Existence of Extremal Solution of the initial value problem

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Abstract: In this paper, we present the existence results along with the locally attractivity and extremal solutions for fractional order non linear functional integro - differential equation in Banach Algebras. We make use of the standard tools of the hybrid fixed Point theory for three operators to establish the main result. We also proved the existence solution are locally attractive in $\mathcal{R}_+$ and extremal result are also proved. Finally, our results are illustrated by a concrete example.

Keywords: - Banach Algebras, Integro-Differential Equation, Existence Result, Fixed Point Theorem, Locally Attractive Solution, Extremal Solution.

1. Introduction:

Fractional Calculus is a generalization of ordinary differential and integration to arbitrary (non-integer) order. The subject has its origin in the 1600s.During three centuries, the theory of fractional calculus developed as a pure theoretical field, useful only for mathematicians. We can see that the tremendous development of theory of fractional calculus occurred in last 3-4 decades. Fractional differentiation proved very useful in various fields of applied sciences and engineering. The "bible" of fractional calculus is the book of Samko, Kilbas and Marichev [34].Several definitions of fractional derivatives and integral are available in the literature, including the Riemann-Liouville, Caputo derivatives and integral [1, 2, 3, 11,13]. The combination of fractional calculus and integral equations may introduce more effective tool for analysis.

2. Statement of the Problem:

Let $\zeta, \delta \in (0,1)$, $\mathcal{R}$ denote the real numbers whereas $\mathcal{R}_+$ be the set of nonnegative numbersi.e. $\mathcal{R}_+ = [0, \infty) \subset \mathcal{R}$

Consider thefractional order nonlinear functional integro-differential equation

$$D^{\zeta}\left[\frac{D^{\delta}x(t) - \sum_{k=1}^{n} I^{\beta_k} h_k(t, x(\gamma(t)))}{f(t, x(\alpha(t)))}\right] = g\left(t, x(\mu(t)), I^{\rho}x(\tau(t))\right) \;\forall\; t \in \mathcal{R}_+ \quad (2.1)$$

$x(0) = 0$ and $D^{\delta}x(0) = 0$

where $0 < \zeta, \delta < 1$, $0 < \zeta + \delta < 2$, $f(t,x) = f : \mathcal{R}_+ \times \mathcal{R} \to \mathcal{R}\setminus\{0\}$, $g(t,x,y) = g : \mathcal{R}_+ \times \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ and $h_k : \mathcal{R}_+ \times \mathcal{R} \to \mathcal{R}$ with $h_k(0,0) = 0$, $k = 1,2,\ldots\ldots n$, $\alpha, \gamma, \tau : \mathcal{R}_+ \to \mathcal{R}_+$.

D^{ζ} denotes R-L fractional derivative of order ζ and I^{ρ} denotes R-L fractional integral of order ρ.

By a solution of the (2.1) we mean a function $x \in BC(\mathcal{R}_+, \mathcal{R})$ that satisfies (2.1) on $\mathcal{R}_-$. Where $BC(\mathcal{R}_-, \mathcal{R})$is the space of continuous and bounded real-valued functions defined on $\mathcal{R}_+$.

Applying a hybrid fixed point theorem [5], the existence results forFIDE (2.1) will be obtained.

In section 3 we recall some useful preliminaries. while in section 4 we deal with the existence of extremal solution of the initial value problem (2.1).Example illustrating the obtained results are presented in section 5.

3. Preliminaries:

Let $X = BC(\mathcal{R}_+, \mathcal{R})$be Banach algebra with norm $\|.\|$ and letΩ be a subset of X. Let a mapping $\mathcal{A} : X \to X$ be an operator and consider the following operator equation in X, namely,

$$x(t) = (\mathcal{A}x)(t), \quad t \subset \mathcal{R}_+ \tag{3.1}$$

Below we give different characterizations of the solutions for operator equation (3.1) on $\mathcal{R}_+$.

We list some precise definitions in the sequel.

Definition 3.1[22]: The solution $x(t)$ of the equation (3.1) is said to be locally attractive if there exists an closed ball $B_r[0]$ in $BC(\mathcal{R}_+, \mathcal{R})$ such that for arbitrary solutions $x = x(t)$ and $y = y(t)$ of equation (3.1) belonging to $B_r[0] \cap \Omega$ suchthat

$$\lim_{t \to \alpha} (x(t) - y(t)) = 0 \tag{3.2}$$

Definition 3.2[22]: Let Xbe a Banach space. A mapping $\mathcal{A} : X \to X$ is called Lipschitz if there is a constant $\alpha > 0$ such that$\|\mathcal{A}x - \mathcal{A}y\| \le \alpha\|x - y\|$ for all $x, y \in X$.
If $\alpha < 1$ then $\mathcal{A}$ is called a contraction on X with the contraction constant α.

Definition 3.3: (Dugundji and Granas [18]). An operator $\mathcal{A}$ on a Banach space X into itself is called Compact if for any bounded subset S of X, $\mathcal{A}$(S) is a relatively compact subset of X. If $\mathcal{A}$ is continuous and compact, then it is called completely continuous on X.
Let Xbe a Banach space with the norm $\|.\|$and Let $\mathcal{A} : X \to X$ be an operator (in general nonlinear). Then $\mathcal{A}$ is called

(i) Compact if $\mathcal{A}(X)$ is relatively compact subset of X;
(ii) Totally bounded if $\mathcal{A}(S)$is a totally bounded subset of X for any bounded subset S of X
(iii) Completely continuous if it is continuous and totally bounded operator on X.

It is clear that every compact operator is totally bounded but the converse need not be true.

The solutions of (2.1) in the space $BC(\mathcal{R}_+,\mathcal{R})$of continuous and bounded real-valued functions defined on $\mathcal{R}_-$. Define a standard supremum norm $\|.\|$ and a multiplication "." in $BC(\mathcal{R}\ ,\mathcal{R})by\|x\| = \sup\{|x(t)|: t \in \mathcal{R}_+\}$(3.3)

$(xy)(t) = x(t)y(t) \quad t \in \mathcal{R}_+$ (3.4)

Clearly, $BC(\mathcal{R}_+,\mathcal{R})$becomes a Banach space with respect to the above norm and the multiplication in it. By $\mathcal{L}^1(\mathcal{R}_+,\mathcal{R})$ we denote the space of Lebesgueintegrable functions on $\mathcal{R}_+$ with the norm$\|.\|_{\mathcal{L}^1}$ defined by

$$\|x\|_{\mathcal{L}^1} = \int_0^\infty |x(t)|dt \tag{3.5}$$

Denote by $\mathcal{L}^1(a,b)$be the space of Lebesgueintegrable functions on the interval (a, b), which is equipped with the standard norm. Let $x \in \mathcal{L}^1(a,b)$ and let $\beta > 0$ be a fixed number.

Definition 3.4[21]: The Riemann-Liouville fractional integral of order $\beta$of the function $x(t)$ is defined by the formula:

$$I^\beta x(t) = \frac{1}{\Gamma(\beta)}\int_0^t \frac{x(s)}{(t-s)^{1-\beta}}ds \qquad t \in (a,b) \tag{3.6}$$

Where $\Gamma(\beta)$ denote the gamma function.

It may be shown that the fractional integral operator I^β transforms the space $\mathcal{L}^1(a,b)$ into itself and has some other properties (see [12-19])

Definition 3.5: Aset A$\subseteq$ [a,b] is said to be measurable if $m^* A = m_* A$. In this case we define mA , the measure of A as $mA = m^* A = m_* A$

If A_1 and A_2 are measurable subsets of [a ,b] then their union and their intersection is also measurable.

Clearly every open or closed set in R is measurable.

Definition 3.6: Let f be a function defined on [a,b].Then f is measurable function if for each $\alpha \in R$, $the set$ $\{x: f(x) > \alpha\}$ is measurable set .

i.e. f is measurable function if for every real number α the inverse image of (α,∞) is an open set

As (α,∞) is an open set and if f is continuous, then inverse image under $f$of $(\alpha,\infty)$is Open sets being measurable , hence every continuous function is measurable.

Definition 3.6:A sequence of functions $\{f_n\}$is said to converge uniformly on an interval $[a,b]$ to a function f if for any $\epsilon > 0$ and for all $x \in [a,b]$ there exists an integer N (dependent only on ϵ) such that for all $x \in [a,b]$

$$|f_n(x) - f(x)| < \epsilon \quad \forall n \geq N$$

Definition 3.7: The Family F is equicontinuous at a point $x_0 \in X$ if for every $\varepsilon > 0$ there exists $\delta > 0$ such that $d(f(x_0), f(x)) < \varepsilon$ for all $f \in F$ and all x that $d(x_0, x) < \delta$.
The family is point wiseequicontinuous if it is equicontinuous at each pointof X.
The family is uniformly equicontinuous if for every $\varepsilon > 0$ there exists $\delta > 0$ a such that $d(f(x_1), f(x_2)) < \varepsilon$ for all $f \in F$ and all $x_1, x_2 \in X$ such that $d(x_1, x_2) < \delta$.
Lemma 3.1 [17]: Let $q > 0$ *and* $x \in C(0,T) \cap \mathcal{L}(0,T)$ Then we have

$$I^q \frac{d^q}{dt^q} x(t) = x(t) - \sum_{j=1}^{n} \frac{(I^{n-q}x)^{(n-j)}(0)}{\Gamma(q-j+1)} t^{q-j}$$

,where $n - 1 < q < n$.

4. Existence of Extremal Solution:

In this section we consider the following Definitions and show that given equation (2.1) has Maximal and Minimal solution:

Definition 4.1 : (Chandrabhan) A function $f: \mathcal{R}_{-} \times \mathcal{R} \to \mathcal{R}$ is called chandrabhan if

i) The function $(x, y) \to f(x, y, z)$ is measurable for each $z \subset \mathcal{R}$
ii) The function $z \to f(x, y, z)$ is non-decreasing for almost each $(x, y) \in \mathcal{R}_+$

Definition 4.2: A function $p_1 \in BC(\mathcal{R}_+, \mathcal{R})$ is called a **lower solution** of the FQFIDE (2.1) on $\mathcal{R}_+$ if the function

$$t \to \left\{\frac{D^\delta p_1(t) - \sum_{k=1}^{n} I^{\beta_k} h_k(t, p_1(\gamma(t)))}{f(t, p_1(\alpha(t)))}\right\}$$ is continuous absolutely and

$$p_1(t) \le I^\delta f(t, x(\alpha(t))\frac{1}{\Gamma(\zeta)}\int_0^t \frac{g\left(t, x(\mu(t)), I^\rho x(\tau(t))\right)}{(t-s)^{1-\zeta}} ds + \sum_{k=1}^{n} I^{\beta_k+\delta} h_k(t, x(\gamma(t)) \quad (4.1)$$

Again a function $p_2 \in BC(\mathcal{R}_+, \mathcal{R})$ is called an **upper solution** of the FQFIDE (2.1) on $\mathcal{R}_+$ if the function

$$t \to \left\{\frac{D^\delta p_2(t) - \sum_{k=1}^{n} I^{\beta_k} h_k(t, p_2(\gamma(t))}{f(t, p_2(\alpha(t))}\right\}$$

is continuous absolutely and

$$p_2(t) \geq I^{\delta} f(t, x(\alpha(t)) \frac{1}{\Gamma(\zeta)} \int_0^t \frac{g\left(t, x(\mu(t)), I^{\rho} x(\tau(t))\right)}{(t-s)^{1-\zeta}} ds$$
$$+ \sum_{k=1}^{n} I^{\beta_k + \delta} h_k(t, x(\gamma(t)) \qquad (4.2)$$

Definition 4.3 [10,35]: A closed and non-empty set $\mathcal{K}$ in a Banach Algebra X is called a cone if

i. $\mathcal{K} + \mathcal{K} \subseteq \mathcal{K}$
ii. $\lambda\mathcal{K} \subseteq \mathcal{K}$ for $\lambda \in \mathcal{K}, \lambda \geq 0$
iii. $\{-\mathcal{K}\} \cap \mathcal{K} = 0$ where 0 is the zero element of X.

and is called **positive cone** if

iv. $\mathcal{K} \circ \mathcal{K} \subseteq \mathcal{K}$

and the notation $\circ$ is a multiplication composition in X

We introduce an order relation $\leq$ in X as follows.

Let $x, y \in X$ then $x \leq y$ if and only if $y - x \in \mathcal{K}$. A cone $\mathcal{K}$ is called normal if the norm $\|\cdot\|$ is monotone increasing on $\mathcal{K}$. It is known that if the cone $\mathcal{K}$ is normal in X then every order-bounded set in X is norm-bounded set in X.

Definition 4.4 : A solution x_M of the Integral equation is said to be maximal if for any other solution x to the problem $x(t) \leq x_M(t) \forall t \in \mathcal{R}$

Again a solution x_m of the Integral equation is said to be Integral equation if for any other solution x to the problem $x_m(t) \leq x(t) \forall t \in \mathcal{R}$

Lemma 4.1[13]: Let $p_1, p_2, q_1, q_2 \in \mathcal{K}$ be such that $p_1 \leq q_1$ and $p_2 \leq q_2$ then $p_1 p_2 \leq q_1 q_2$.

For any $p_1, p_2 \in X = \mathcal{C}(\mathcal{R}_+, \mathcal{R}), p_1 \leq p_2$ the order interval $[p_1, p_2]$ is a set in X given by. $[p_1, p_2] = \{x \in X : p_1 \leq x \leq p_2\}$

Definition 4.5[6]: A mapping $R: [p_1, p_2] \to X$ is said to be nondecreasing or monotone increasing if $x \leq y$ implies $Rx \leq Ry$ for all $x, y \in [p_1, p_2]$.

Theorem 4.1[14]: Let $\mathcal{K}$ be a cone in Banach Algebra X and let $[p_1, p_2] \in X$. Suppose that $\mathcal{A}, \mathcal{B}: [p_1, p_2] \to \mathcal{K}$ and $\mathcal{C}: [p_1, p_2] \to X$ be three nondecreasing operators such that

a. $\mathcal{A}$ and $\mathcal{C}$ are a Lipschitz with Lipschitz constant α, β
b. $\mathcal{B}$ is completely continuous,

c. The elements $p_1, p_2 \in X$ satisfy $p_1 \leq \mathcal{A}p_1\mathcal{B}p_1 + \mathcal{C}p_1$ and $\mathcal{A}p_2\mathcal{B}p_2 + \mathcal{C}p_2 \leq p_2$ Further if the cone $\mathcal{K}$ is normal and positive then the operator equation $x = \mathcal{A}x\mathcal{B}y + \mathcal{C}$ has the least and greatest positive solution in $[p_1, p_2]$ whenever $\alpha M + \beta < 1$, where $M = \|\mathcal{B}([p_1, p_2])\| = sup\{\|\mathcal{B}x\| : x \in [p_1, p_2]\}$.

we consider another hypothesis

(H_1)The function $x \to \left\{\frac{D^{\delta}x(t) - \sum_{k=1}^{n} I^{\beta_k} h_k(t, x(\gamma(t)))}{f(t, x(\alpha(t))}\right\}$ is increasing in the interval

$[min_{t\in\mathcal{R}_+} p_1(t), max_{t\in\mathcal{R}_+} p_2(t)]$.

(H_2)The FQFIDE (2.1) has a lower solution p_1 and upper solution p_2 on $\mathcal{R}_+$ with $p_1 \leq p$

(H_3)The function g is caratheodory.

(H_4)The functions $f: \mathcal{R}_+ \times \mathcal{R} \to \mathcal{R} - \{0\}, g: \mathcal{R}_+ \times \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ and $h_k: \mathcal{R}_+ \times \mathcal{R} \to \mathcal{R}$ a nondecreasing in x almost everywhere for $t \in \mathcal{R}_+$.

(H_5)The function $m_1: \mathcal{R}_+ \to \mathcal{R}$ defined by

$$m_1(t) = \left|g\left(t, p_1(\mu(t)), I^{\rho}p_1(\tau(t))\right)\right| + \left|g\left(t, p_2(\mu(t)), I^{\rho}p_2(\tau(t))\right)\right|$$

is Lebesgue measurable.

Remark 4.1: Assume that the hypotheses $((H_1)-(H_5))$ holds, then the function $t \to g\left(t, x(\mu(t)), I^{\rho}x(\tau(t))\right)$ is lebesgue integrable on $\mathcal{R}_+$, say

$\left|g\left(t, x(\mu(t)), I^{\rho}x(\tau(t))\right)\right| \leq m_1(t)$, a.e., $t \in \mathcal{R}_+$

For all $x \in [p_1, p_2]$ and some Lebesgue integrable function m_1.

Theo 4.2 : Suppose that the Hypothesis$((H_1) - (H_5))$ are holds and

$\|\alpha\| \left\{\frac{1}{\Gamma(\zeta+1)} T^{\zeta} \|m_1\|_{\mathcal{L}^1}\right\} + \|\beta\| < 1$.Thenproblem (2.1) hasa minimal and maximal positive solutions on $\mathcal{R}$.

Proof : Let $X = C(\mathcal{R}_+, \mathcal{R})$ and we define an order relation "$\leq$" by the cone $\mathcal{K}$ given by (5.3). Clearly $\mathcal{K}$ is a normal cone in X. Define three operators $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ on X by (4.2), (4.3) and (4.4) respectively. Then FQFIDE (2.1) is transformed into an operator

(4.2), (4.3) and (4.4) respectively. Then FQFIDE (2.1) is transformed into an operator equation $\mathcal{A}x\mathcal{B}x + \mathcal{C}x = x$in Banach algebra X. Notice that (H_6) implies $\mathcal{A}, \mathcal{B}: [p_1, p_2] \rightarrow \mathcal{K}$ also note that (H_7) ensures that $p_1 \leq \mathcal{A}p_1\mathcal{B}p_1 + \mathcal{C}p_1$ and $\mathcal{A}p_2\mathcal{B}p_2 + \mathcal{C}p_2 \leq p_2$. Since the cone $\mathcal{K}$ in X is normal, $[p_1, p_2]$is a norm bounded set in X. Now it is shown, as in the proof of Theorem (2.1), that $\mathcal{A}$ and $\mathcal{C}$ are Lipschitz with a Lipschitz constant $\|\alpha\|$ and $\|\beta\|$ respectively. Similarly $\mathcal{B}$ is completely continuous operator on $[p_1, p_2]$. Again the hypothesis (H_9) implies that $\mathcal{A}, \mathcal{B}$and $\mathcal{C}$ are non-decreasing on $[p_1, p_2]$. To see this, let $x_1, x_2 \in [p_1, p_2]$ be such that $x_1 \leq x_2$. Then by (H_9),

$$x(t) = I^{\delta} f(t, x(\alpha(t)) \frac{1}{\Gamma(\zeta)} \int_0^t \frac{g\left(t, x(\mu(t)), I^{\rho} x(\tau(t))\right)}{(t-s)^{1-\zeta}} ds + \sum_{k=1}^{n} I^{\beta_k + \delta} h_k(t, x(\gamma(t))$$

$$\mathcal{A}x_1(t) = I^{\delta} f(t, x_1(\alpha(t)) \leq I^{\delta} f(t, x_2(\alpha(t)) = \mathcal{A}x_2(t) \quad \text{for all } t \in \mathcal{R}_+$$

and

$$\mathcal{B}x_1(t) = \frac{1}{\Gamma(\zeta)} \int_0^t \frac{g\left(t, x_1(\mu(t)), I^{\rho} x_1(\tau(t))\right)}{(t-s)^{1-\zeta}} ds$$

$$\leq \left[\frac{1}{\Gamma(\zeta)} \int_0^t \frac{g\left(t, x_2(\mu(t)), I^{\rho} x_2(\tau(t))\right)}{(t-s)^{1-\zeta}} ds\right] = \mathcal{B}x_2(t)$$

$$\mathcal{C}x_1(t) = \sum_{k=1}^{n} I^{\beta_k + \delta} h_k(t, x_1(\gamma(t)) \leq \sum_{k=1}^{n} I^{\beta_k + \delta} h_k(t, x_2(\gamma(t)) \leq \mathcal{C}x_2(t), \qquad t \in \mathcal{R}_+$$

So,$\mathcal{B}$ *and* $\mathcal{C}$ are non decreasingoperatorson$[x_1, x_2]$

Again by Hypothesis (H_7)

$$p_1(t) \leq I^{\delta} f(t, p_1(\alpha(t)) \frac{1}{\Gamma(\zeta)} \int_0^t \frac{g\left(t, p_1(\mu(t)), I^{\rho} p_1(\tau(t))\right)}{(t-s)^{1-\zeta}} ds$$
$$+ \sum_{k=1}^{n} I^{\beta_k + \delta} h_k(t, p_1(\gamma(t))$$

$$\leq I^{\delta} f(t, x(\alpha(t))\frac{1}{\Gamma(\zeta)}\int_0^t \frac{g\left(t, x(\mu(t)), I^{\rho}x(\tau(t))\right)}{(t-s)^{1-\zeta}} ds + \sum_{k=1}^{n} I^{\beta_k+\delta} h_k(t, x(\gamma(t))$$

$$\leq I^{\delta} f(t, p_2(\alpha(t))\frac{1}{\Gamma(\zeta)}\int_0^t \frac{g\left(t, p_2(\mu(t)), I^{\rho}p_2(\tau(t))\right)}{(t-s)^{1-\zeta}} ds + \sum_{k=1}^{n} I^{\beta_k+\delta} h_k(t, p_2(\gamma(t))$$

$\leq p_2(t), \forall t \in \mathcal{R}_+$ and $x \in [p_1, p_2]$

As a result $p_1(t) \leq \mathcal{A}x(t)\mathcal{B}x(t) + \mathcal{C}x(t) \leq p_2(t), \forall t \in \mathcal{R}_+$ and $x \in [p_1, p_2]$

Hence $\mathcal{A}x\mathcal{B}x + \mathcal{C}x \in [p_1, p_2] \forall x \in [p_1, p_2]$

Again $M = \|\mathcal{B}([p_1, p_2])\| = sup\{\|\mathcal{B}x\| : x \in [p_1, p_2]\}$

$$\leq sup\left\{ sup_{t\in\mathcal{R}_+}\left\{\frac{1}{\Gamma(\zeta)}\int_0^t \frac{\left|g\left(t, x(\mu(t)), I^{\rho}x(\tau(t))\right)\right|}{(t-s)^{1-\zeta}}\right\} : x \in [p_1, p_2]\right\}$$

$$\leq sup\left\{\frac{1}{\Gamma(\zeta)}\left[\frac{(t-s)^{\zeta}}{\zeta}\right]_0^t \|m_1\|_{\mathcal{L}^1}\right\} \leq \frac{1}{\Gamma(\zeta+1)} T^{\zeta} \|m_1\|_{\mathcal{L}^1}$$

Since $\alpha M + \beta < \|\alpha\| \left\{\frac{1}{\Gamma(\zeta+1)} T^{\zeta} \|m_1\|_{\mathcal{L}^1}\right\} + \|\beta\| < 1$

Thus by theorem (4.2) given fractional order nonlinrar functional integro-differential equation (2.1) has a minimal and maximal positive solutions on $\mathcal{R}$.

5 Example: Consider the following fractional order quadratic functional ID equation of type (2.1)

$$D^{1/2}\left[\frac{D^{2/3}x(t) - \sum_{k=1}^{3} I^{\beta_k} h_k(t, x(\gamma(t)))}{f\left(t, x(\alpha(t))\right)}\right] = g\left(t, x(\mu(t)), I^{\rho}x(\tau(t))\right) \forall t \in \mathcal{R}_+ \quad (6.1)$$

$x(0) = 0$ and $D^{2/3}x(0) = 0$

$$f\left(t, x(\alpha(t))\right) = (\sin(\pi t + 2t))\left\{\frac{|x(t)| - 2}{|x(t)| + 5} + \frac{t-8}{15}\right\}$$

$$g\left(t, x(\mu(t)), I^{\rho} x(\tau(t))\right) = \frac{t}{\frac{1}{4} e^{t} \sin\left\{\frac{4|x|}{2+|x|} \cos\frac{\pi t}{8}\right\} - \frac{1}{4} e^{t} \sin\left\{\frac{4 I^{11/3}|x|}{2 + I^{11/3}|x|} \cos\frac{\pi t}{8}\right\}}$$

$$\sum_{k=1}^{3} I^{\beta_k} h_k(t, x(\gamma(t)))$$

$$= I^{1/4}\left(2 + \frac{e^{t}\sin 4t}{1 + |x(t)|}\right) + I^{1/7}\left(\frac{\sin x(t)\cos t}{t+2} + \frac{1}{t}\right) + I^{7/4}\left(\frac{|x(t)| t e^{t}}{1 + e^{t}}\right)$$

$Here\ \zeta = \frac{1}{2}, \delta = \frac{2}{3}\ k = 3\ \beta_1 = \frac{1}{4}, \beta_2 = \frac{1}{7}, \beta_3 = \frac{7}{4}, \rho = \frac{11}{3}, \alpha = \pi t + 2t, \mu = t, \tau = \pi t$

$obiviously\ \alpha = \pi t + 2t,\ \mu = t,\ \tau = \pi t$ are continuous.

Hence the entire hypotheses are satisfied. Consequently all the conditions of theorem (2.3) are satisfied.

Conclusion :- In this paper we have studied the existence and locally attractively of solution to the second order nonlinear quadratic functional differential equation in banach space by fixed point theory,

Thus problem (5.1) has at least one solution on $t \in \mathcal{R}_+$

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Analytical study of Mellin-Stieltjes transform

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Abstract :-

This present paper deals with Laplace integral transform, define analytic function, million transform stieltjes resultant of two function and non negative constant C and resultant of indefinite integrals and kernels semi definite quadratic forms.

Integral transform are applied and helpful in the solution of partial differential equation. But, the choice of a particular transform to be used for the solution of a differential equation depends on the characteristic of the boundary condition of the equation.

Keywords:- Laplace integral transform, Analytic function, Mellin transform, Stieltjes transform,Stieltjes resultant of two function

Introduction: The Laplace transform is very profitable in investigate and plan for the systems that are linear and invariant. In the beginning of 1910, these transform techniques were applied in signal processing at bell labs for the signal filtering and telephone long lines communication by H.Bode and others. After while transform theory provided the backbone of classical control theory. These were practiced during the time of world wars and up to about 1960. At that the time of 1960 the state variable techniques began to be used for control designs. A French mathematician Pierre-Simon Laplace (1749-1827) characterized by the fenchsevolution. The name Laplace transform is derived from the French mathematician and astronomer Pierre-Simon Laplace, who used similar type of transform theory in his work on probability theory. The transform theory was widespread at the time after world war 2nd. And in 19th century it is used by Abel, lerch, Heaviside and Bronwich.

The Laplace transform is also related to the fourier transform, but the fourier transform tells about a function or signal as a series of modes of vibrations, where as the Laplace transform tells about a function into its moments. Like the fourier transform, the Laplace transform is used for solve the differential equation and integral equations. In physics and engineering the Laplace transform is used for analysis of linear time invariant system such as electrical circuits, harmonic oscillator's optical devices and mechanic system. In the kind of analysis, the Laplace transform is often assumed as a transform from the time domain, in which inputs and outputs are functions of complex angular frequency.

1.1 Defines an analytic function

Now, extending the outcome of integral transform.

Axiom 1.1(a) If condition for $k < 1$

$$\int_0^\infty e^{-2kt}|\phi(t)|^2 dt < \infty$$

$$\int_{-\infty}^0 e^{-2lt}|\phi(t)|^2 dt < \infty,$$

Then;

$$\underset{R\to\infty}{\overset{(2)}{\text{l.i.m.}}} \int_{-R}^{R} e^{-st}\phi(t)dt$$

exists for **k≤σ≤l condition** of an analytic function f(s) for $k < \sigma < l$

$$f(s) = \int_{-\infty}^{\infty} e^{-st}\phi(t)dt,$$

the integral **converging** absolutely for $k < \sigma < l$, and

$$\underset{\sigma\to k+}{\overset{(2)}{\text{l.i.m.}}} f(\sigma + i\tau) = f(k + i\tau)$$

$$\underset{\sigma\to l-}{\overset{(2)}{\text{l.i.m.}}} f(\sigma + i\tau) = f(l + i\tau).$$

As mention here, the notation l.i.m. signifies mean square convergence. Thus,

$$\underset{R\to\infty}{\overset{(2)}{\text{l.i.m.}}} F_R(s) = F(s)$$

for $k \leq \sigma \leq l$ means that for any such σ

$$\lim_{R\to\infty} \int_{-\infty}^{\infty} \left|F_R(\sigma + i\tau) - F(\sigma + i\tau)\right|^2 d\tau = 0.$$

Axiom 1.1(b) If function f(s) is mentioned & explained in Theorem 1.1 a, then-

$$\underset{T\to\infty}{\overset{(2)}{\text{l.i.m.}}} \frac{e^{-ct}}{2\pi i} \int_{c-iT}^{c+iT} f(s)e^{st}ds = \phi(t)e^{-ct} \qquad (k \leq c \leq l)$$

And,

$$\int_{-\infty}^{\infty} e^{-2\sigma t}|\phi(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |f(\sigma + i\tau)|^2 d\tau \qquad (k \leq \sigma \leq l).$$

This outcome provides a direct consequence of the Plancherel theorem,

1.2 The Mellin Transform

This is the transform

$$f(s) = \int_0^\infty t^{s-1}\psi(t)dt$$

which was formulated & applied by Riemann and Cahen. But, it was first put on a rigorous basis by H. Mellin [1902] and now known with his name. It may be obtained by an exponential transformation from the integral Laplace transform and hence, no special treatment is required. It is thus, useful for reference to have the outcomes recorded in the Mellin form.

In the following integral,

$$f(s) = \int_{-\infty}^\infty e^{-st} d\alpha(t) \qquad (1.2,1)$$

we make the change of variable $e^{-t} = u$ and obtain

$$f(s) = \int_{0+}^\infty u^s d\beta(u) \qquad (1.2,2)$$

Where;

$$\beta(u) = -\alpha(\log u^{-1}).$$

Referring to (1.2,2) as the Mellin-Stieltjes transform. If $\alpha(t)$ is an integral of $\phi(t)$ the integral (1.2,2) develops-

$$f(s) = \int_{0+}^\infty u^{s-1}\psi(u)du, \qquad (1.2,3)$$

Where,

$$\psi(u) = \phi(\log u^{-1}).$$

As mentioned above, its the classical form of the Mellin transform with an exception that we are here taking the integral as a Cauchy limit at its lower limit of integration.

Axiom 1.2(a) If the integral

$$\psi(s)=\int_0^\infty t^{s-1}\psi(t)dt \qquad (1.2,4)$$

converges actually on the line $\sigma = c$, and if $\psi(t)$ is of closely-related variation in a neighbourhood of $t = x(x > 0)$, then-

$$\lim_{T\to\infty}\frac{1}{2\pi i}\int_{c-iT}^{c+iT} f(s)x^{-s}ds = \frac{\psi(x+)+\psi(x-)}{2}.$$

This follows from Theorem. Since $t^{c-1}\psi(t)$ belongs to L on $(0,\infty)$ we may clearly substituting (1.2,3) by (1.2,4.

Axiom 1.2(b) If the integral (1.2,2) converges for $\sigma_c' < \sigma < \sigma_c''$, then,

$$\lim_{T\to\infty}\frac{1}{2\pi i}\int_{c-iT}^{c+iT} \frac{f(s)t^{-2}}{s}ds = \beta(\infty)-\frac{\beta(t+)+\beta(t-)}{2} \qquad (c>0)$$

$$=\beta(0+)-\frac{\beta(t+)+\beta(t-)}{2} \qquad (c<0)$$

for value of $c \neq 0$ between σ_c' and σ_c''.

This is similar of Axiom.

Axiom 1.2(c) When k<1 & if $u^k\psi(u)$ relates to L^2 on (0, 1) and $u^l\psi(u)$ to L^2 on $(1,\infty)$, then-

$$\underset{R\to\infty}{\overset{(2)}{\text{l.i.m.}}}\int_{R^{-1}}^{R} u^{s-1}\psi(u)du$$

for $k \leq \sigma \leq l$ and explains a function f(s) which is analytic in the strip $k < \sigma < l$. However;

$$f(s)=\int_0^\infty u^{s-1}\psi(u)du \qquad (k < \sigma < l),$$

the integral converging absolutely, and

$$\underset{\sigma\to k+}{\overset{(2)}{(\text{l.i.m.})}} f(\sigma+i\tau)=f(k+i\tau)$$

$$\lim_{T\to\infty}\int_0^\infty \left|\psi(u)u^c - \frac{u^c}{2\pi i}\int_{c-i\tau}^{c+i\tau} f(s)u^{-s}ds\right|^2 \frac{du}{u} = 0 \qquad (k \leq c \leq 1)$$

and-

$$\int_0^\infty u^{2c-1}|\psi(u)|^2 du = \frac{1}{2\pi}\int_{-\infty}^{\infty}|f(c+i\tau)|^2 d\tau \qquad (k \leq c \leq 1).$$

Mentioned above two theorems are connecting bands of theorem. Particularly, when taken c=1/2 we observed the following;

$$\frac{1}{2\pi i}\int_{1/2-iT}^{1/2+iT} f(s)u^{-s}ds$$

As a result, it changes in the mean to $\psi(u)$ in the interval $(0,\infty)$.

1.3 Stieltjes Outcome

Assuming $\alpha(t)$ and $\beta(t)$ be two functions known for all real value of t.

Now, defining Stieltjes outcome.

DEFINITION. The Stieltjes outcome of $\alpha(t)$ and $\beta(t)$ is the function

$$\gamma(t) = \int_{-\infty}^{\infty}\alpha(t-u)d\beta(u) = \int_{-\infty}^{\infty}\beta(t-u)d\alpha(u) \qquad (1.3,5)$$

If the above mention integral exists & hence found to be equal then

$\alpha(t)$ is continuous for $-\infty < t < \infty$, if

$$\int_{-\infty}^{\infty}|d\beta(u)| < \infty, \ \beta(-\infty) = 0,$$

If condition applied $\alpha(\infty), \alpha(-\infty)$ exist with $\alpha(-\infty) = 0$, then the integrals (1.3,5) exist and are found equal. Hence, outcome of $\alpha(t)$ and $\beta(t)$ exists. Among the given pairs of end conditions would be enough. :

$$\lim_{T\to\infty}\int_0^{\infty}\left|\psi(u)u^c-\frac{u^c}{2\pi i}\int_{c-i\tau}^{c+i\tau}f(s)u^{-s}ds\right|^2\frac{du}{u}=0 \qquad (k\leqq c\leqq 1)$$

and-

$$\int_0^{\infty}u^{2c-1}|\psi(u)|^2du=\frac{1}{2\pi}\int_{-\infty}^{\infty}|f(c+i\tau)|^2d\tau \qquad (k\leqq c\leqq 1).$$

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$\alpha(t)$ is continuous for $-\infty<t<\infty$, if

$$\int_{-\infty}^{\infty}|d\beta(u)|<\infty,\ \beta(-\infty)=0,$$

If condition applied $\alpha(\infty),\alpha(-\infty)$ exist with $\alpha(-\infty)=0$, then the integrals (1.3,5) exist and are found equal. Hence, outcome of $\alpha(t)$ and $\beta(t)$ exists. Among the given pairs of end conditions would be enough. :

$$\begin{cases}\alpha(-\infty)=0\\ \alpha(\infty)=0,\end{cases}\quad \begin{cases}\alpha(-\infty)=0\\ \beta(-\infty)=0,\end{cases}\quad \begin{cases}\beta(\infty)=0\\ \alpha(\infty)=0,\end{cases}\quad \begin{cases}\beta(\infty)=0\\ \beta(-\infty)=0,\end{cases}$$

According to our assumption on the right-hand side of (1.3,5) it is the Cauchy value of the integral which is known to exist. It need not change actually. Instead by symmetry the conditions implied on $\alpha(t)$ and $\beta(t)$ may be alternative.

Now, given case emphasizes that both $\alpha(t)$ and $\beta(t)$ follows bounded variation in $(-\infty,\infty)$.

$$\int_{-\infty}^{\infty}|d\alpha(t)|<\infty,\qquad \int_{-\infty}^{\infty}|d\beta(t)|<\infty \tag{1.3,6}$$

Then, $\alpha(\pm\infty)$ and $\beta(\pm\infty)$ endures that no significant limitation in assuming the below equation $\alpha(-\infty)=\beta(-\infty)=0$.

Assuming P_α be the countable set of points where $\alpha(t)$ is not continuous (probably a null set), and P_β the set where $\beta(t)$ is not continuous.

To proof:

Axiom 1.3(a) Let $\alpha(t)$and $\beta(t)$ are of closely-related variation in $(-\infty,\infty)$ without continuities in P_α and P_β simultaneously, and if $\alpha(-\infty)=\beta(-\infty)=0$, then the Stieltjes outcome $\gamma(t)$of $\alpha(t)$ and $\beta(t)$ endures for all t not in $P_{\alpha+\beta}$.

For, if t is absent in $P_{\alpha+\beta}$, then $\alpha(t-u)$and $\beta(u)$ can have a mutual point of continuity for any u in $(-\infty,\infty)$. Hence, for t absent in $P_{\alpha+\beta}$ the integrals

$$\int_S^R \alpha(t-u)d\beta(u),\qquad \int_S^R \beta(t-u)d\alpha(u)$$

exist for every R and S. Limits of these integrals exists as R becomes positively infinite and S becomes negatively non-finite, so;

$$\int_{-\infty}^{\infty}|\alpha(t-u)||d\beta(u)|<\infty,\qquad \int_{-\infty}^{\infty}|\beta(t-u)||d\alpha(u)|<\infty.$$

$\alpha(t)$and $\beta(t)$are bounded follows from the inequalities

$$|\alpha(t)| \leqq \int_{-\infty}^{t} |d\alpha(u)|, \qquad |\beta(t)| \leqq \int_{-\infty}^{t} |d\beta(u)|$$

and (1.3,6. Lastly the two integrals (1.3,5) are equal by using integration by parts.

Note: If $\alpha(-\infty)=\beta(-\infty)=0$.then; these 2 integral may exist & be unequal.

This happens, for example, if

$$\alpha(t)=1 \qquad (-\infty<t<\infty)$$

$$\beta(t)=e^{-t} \qquad (t\geq 0)$$

$$=1 \qquad (t<0).$$

There is a difference by unity for the existence of both the integrals.

1.4 Stieltjes Resultant of two functions

To study the nature of the outcome of two functions of bounded variation for variable's large values.

Axiom 1.4(a) When $\alpha(t)$ and $\beta(t)$ proves the conditions of Axiom 1.3(a), then

$$\int_{-\infty}^{\infty} \alpha(t-u)d\beta(u) \to \alpha(\infty)\beta(\infty) \qquad (t\to+\infty)$$

$$\to 0 \qquad (t\to-\infty),$$

the variable t to become non-finite when the set is complementary to $P_{\alpha+\beta}$.

Considering t equals to positively non-finite. Clearly, from the assumptions 7.10(a) that $\beta(\infty)$ and $\alpha(\infty)$ exist. Assume $V_\beta(t)$ be the variation of $\beta(u)$in $-\infty<u\leqq t$. Hence, now its clearly enough to prove that-

$$\lim_{t\to\infty} \int_{-\infty}^{\infty} [\alpha(t-u)-\alpha(\infty)]d\beta(u)=0$$

or that

$$\lim_{t\to\infty} \int_{-\infty}^{\infty} |\alpha(t-u)-\alpha(\infty)|dV_\beta(u)=0 \qquad (1.4,7)$$

Taking $\in$ be an absolute non-negative numeric. Determining R so generous that for all t absent in $P_{\alpha+\beta}$

$$\int_R^\infty |\alpha(t-u)-\alpha(\infty)|dV_\beta(u)<\varepsilon.$$

This can happen since $\beta(t)$ is of bounded variation and $\alpha(t)$ is bounded. Next to select t_0 such that for all t not in $P_{\alpha+\beta}$ but greater than t_0

$$|\alpha(t-u)-\alpha(\infty)|<\varepsilon \qquad (-\infty<u\leqq R).$$

For such t

$$\int_{-\infty}^R |\alpha(t-u)-\alpha(\infty)|dV_\beta(u)<\varepsilon V_\beta(\infty) \tag{1.4,8}$$

From equation (1.4,7) and (1.4,8) we formed the expected result.

The case $t\to-\infty$ may be treated by employing the proof of integral;

$$\int_{-\infty}^\infty [\alpha(u+t)-\alpha(\infty)]d[\beta(\infty)-\beta(-u)]$$

$$=\alpha(\infty)\beta(\infty)+\int_{-\infty}^\infty \alpha(t-u)d\beta(u).$$

COROLLARY. Here, $\alpha(t)$ and $\beta(t)$ are kept close in $(-\infty,\infty)$, such as-

$$\lim \int_{-\infty}^\infty \alpha(t-u)d\beta(u)=\begin{cases}\alpha(\infty)[\beta(\infty)-\beta(-\infty)] & (t\to\infty)\\ \alpha(-\infty)[\beta(\infty)-\beta(-\infty)] & (t\to-\infty)\end{cases}$$

This follows by employing the Axiom to the functions $\alpha(t)-\alpha(-\infty)$ and $\beta(t)-\beta(-\infty)$.

Conclusion

The present paper deals with Laplace integral transform, also consist function of a Laplace integral, summarily defines the analytic function, the mellin transform, stieltjes outcome, stieltjes resultant of two functions, non-negative constant C and representation of non-negative functions, kernels of semi definite quadratic form.

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PROBLEMS MANIFESTED IN PROSPECTIVE SECONDARY MATHEMATICS FUNCTIONS AND COUNTER EXAMPLES IN DIFFERENTIATION

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Abstract

In advanced mathematical thinking, proving and refuting are vital abilities to help demonstrate whether and why a proposition is true or false. Learning proofs and counterexamples in the domain of differentiation is especially important because students encounter differentiation in many mathematics courses. We examined 36 prospective secondary mathematics functions performance producing proofs for statements believed to be true and counterexamples for statements believed to be false. Problems were identified in the teachers' written work, which highlights the need to empower instructors in their teaching and prospective mathematics teachers in their learning to write complete proofs and counterexamples in undergraduate mathematics courses.

Key Word: **Problems Manifested, Mathematics Functions**

INTRODUCTION

Proving and refuting play essential roles in advanced mathematical thinking because they help demonstrate whether and why propositions are true or false. A mathematical proof requires that definitions, statements, or procedures are used to "deduce the truth of one statement from another" helping people understand the logic behind a statement and the "insight into how and why it works". Counterexamples similarly play a significant role in mathematics by illustrating why a mathematical proposition is false; a single counterexample is sufficient to refute a false statement.

Together, mathematical proofs and counterexamples can provide students with insight into meanings behind statements and also help them see why statements are true or false. Accordingly, undergraduates, including prospective mathematics teachers, in advanced mathematics are expected to learn and to use both proofs and counterexamples throughout the undergraduate mathematics curriculum.

In recent years, mathematical proof has received an increased level of attention because of the various roles it plays in the mathematics community, including communication, explanation, or validation of mathematical claims. In fact, the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2010) devoted a standard to reasoning and proof and asserts that students in pre-kindergarten through grade 12 should regularly study mathematical proof.[4]

In order to implement current reform recommendations successfully regarding mathematical proof, prospective teachers must have a solid understanding of proof them. The challenge of fulfilling this demand is that many teachers have not traditionally been expected to teach proof.

Knuth indicated that the enactment of reform efforts with respect to proof in school mathematics depends heavily on teachers' conceptions, "a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences". In light of this challenge, a growing number of researchers have started to investigate conceptions of proof held by pre-service mathematics teachers.

Despite the importance of prospective teachers' conceptions of proof, few research studies have investigated prospective mathematics teachers' abilities in producing proofs and counterexamples in the domain of differentiation, which is essential content across the world in college mathematics. Further, few studies have examined how proofs and counterexamples in differentiation convey the structure of analysis from basic conceptions addressed in previous calculus courses, as well as prospective mathematics teachers' performance proving and refuting statements.

In order to address such deficiencies in the field, the main purpose of this paper is to contribute to the knowledge base in this area by identifying the problems manifested in prospective mathematics teachers' attempts to produce proofs and counterexamples in the mathematical area of differentiation. The findings suggest more attention should be paid to teaching and learning proofs and counterexamples, as prospective mathematics teachers with mathematics majors showed difficulty in writing these statements. More importantly, our analysis also suggests potential means for improving prospective teachers' performance producing proofs and counterexamples in undergraduate mathematics courses.

1. METHODS

Taiwanese undergraduates enrolled in Advanced Calculus I in Fall 2007 at a national university in Taiwan participated in this study. They were selected by convenience sampling; in other words, participants were contacted by colleagues of the researchers and were recruited on the basis of their willingness to participate in the study. Every undergraduate volunteering for the study was accepted, and thus resulted in a sample size of 36. With only a few exceptions, these participants will complete mathematics major and become secondary mathematics teachers.

Differentiation is a topic addressed in their previous calculus courses, thus all of the prospective mathematics teachers participating in this study had some relevant domain knowledge. The instrument, which was comprised of five mathematical statements modified from textbooks and entrance examinations, was designed to provide a measure of prospective teachers' concepts of differentiation.

The instrument was written in English because English is used in advanced calculus courses at the university in Taiwan. Participants were asked to construct proofs for statements they believed to be true and to generate counterexamples for statements they believed to be false.

The errors manifested in the students' attempts to construct proofs and to generate counterexamples were investigated by analyzing participants' written work. Due to page limitations, this paper only focuses on Problem 1, a false statement, and Problem 2, a true statement, as listed in Table 1.

Problem	Mathematical statement	T or F
1.	Let f be a function defined on a set of numbers S, and let $a \in S$. If f is continuous at a, then f is differentiable at a.	False
2.	Let f be a real-valued differentiable function defined on [a, b]. If $f'(a) < \lambda < f'(b)$, then $\lambda = f'(z)$ for some z in (a, b).	True

Table 1: The First Two Propositions Used in This Study

2. RESULTS

A selection of interesting responses to Problems 1 and 2 are discussed here in order to illustrate the prospective mathematics teachers' performance as well as their understandings of differentiation.

Problem 1

With respect to Problem 1 – a false proposition, 1 participant left blank (3%), 7 participants provided an incorrect proof (19%), 2 participants provided no basis for generating a counterexample (6%), 4 participants provided relevant knowledge but not a counterexample (11%), 1 participant provided an incomplete counterexample (3%), and 21 participants provided a complete counterexample (58%).

$$f'(a) = \lim \frac{f(x) - f(a)}{[illegible]}$$

Although the statement in Problem 1 is false, 7 participants of 35 (20%) who answered believed this was true and attempted to provide a proof to support their claim. They all described, "If f is continuous at $a \in S$, then $f(a)$ exists. So exists.

Therefore, f is differentiable at $a \in S$". Such evidence shows that these participants seemed to believe $f'(a)$ exists if f is continuous at $a \in S$ which implies that $f'(a)$ exists.

Moreover, these participants demonstrated an unclear understanding of the differences between continuous functions and differentiation, and thus determined this false proposition to be true. Two participants provided … as a counterexample to refute Problem 1.

One of them indicated that " $f(x)$ is continuous at 0 but $f'(x)$ is not continuous at 0", and the other showed that " $f(x)$ is continuous at -1 but $f'(x)$ is not differentiable at -1". These two participants misrepresented the first derivative of as $\ln x$ which yielded to an invalid counterexample to refute this statement.

Additionally, the first student expressed the misunderstanding of continuity, because f is not continuous at 0. The above descriptions illustrate that these participants did not have a clear understanding of continuous functions as well as differentiation.

Problem 2

As to Problem 2—a true proposition, 8 participants left blank (22%), 6 participants provided an incorrect counterexample (17%), 13 participants provided no basis for constructing a proof (36%), and 9 participants provided relevant knowledge but not a mathematical proof (25%). In addition, no participants produced a complete proof for Problem 2.

While the statement in Problem 2 is true, 6 participants of 28 (21%) who answered believed it to be false and provided an incorrect counterexample. They all provided a polynomial function with a particular domain, and then showed that they found one z which did not belong to a given domain.

For example, one student showed, " $f(x) = x^2 + 3x + 2$ " is a real-valued differentiable function defined on [-1, 2]. $f'(-1) = 1$, $f'(2) = 7$ and let $\lambda = 2$. then $f'(z) = 1 \Rightarrow x = -1 \Rightarrow -1 \notin (-1, 2)$". This student seemed to use a specific example to demonstrate that the proposition was false. Because this student chose $\lambda = 2$, he or she made a mistake on the step of $f'(z) = 2x + 3 = 1 \Rightarrow f'(z) = 1 \Rightarrow 2x = -2 \Rightarrow x = -1$ not belonging to (-1, 2). This result led the student to determine this statement was false.

Four of the participants, who attempted to provide a proof, all described " $f'(a) < f'(z) < f'(b) \Rightarrow f(a) < f(z) < f(b) \Rightarrow a < z < b$ ". They expressed their misunderstandings of the relationships between the differentiation and function as well as the function and the elements and seemed to believe such processes involving symbolic manipulations yield a valid proof. Furthermore, these participants did not show their understandings of what knowledge is appropriate for writing a mathematical proof.

3. DISCUSSION

All participants in this study were mathematics majors, and they had opportunities to learn about differentiation in their current advanced calculus course as well as in previous calculus courses. However, our results suggest that some prospective secondary mathematics functions still possess an inadequate understanding of not only proofs and counterexamples but also differentiation (46% for Problem 1 and 100% for Problem 2 who answered).

Determining the truth or falsity of a proposition and writing a correct proof counterexample requires an understanding of the relevant concepts and strategic knowledge, and some of the participants failed to express such an understanding.

This confirms Weber's findings that undergraduate students did not demonstrate their knowledge techniques of proving. Also, several participants seemed to regard their symbolic computations as a mathematical proof, which is consistent with Harel and Sowder's and Weber's findings.

Overall, the majority of participants did not show a clear understanding of concepts differentiation when producing a proof or counterexample.

CONCLUSIONS

Weber noted that manipulating symbols with understanding, "knowledge of the domain's proof techniques" and "knowledge of which theorems are important and when they will be [illegible] are essential skills in writing mathematical proofs. In order to enhance prospective [illegible] teachers' proof techniques, instructors should provide students with exposure to the same concepts more than once.

Given the mathematical backgrounds of the participants, it is surprising that they still had considerable difficulty identifying the true or false proposition as well as producing proofs and counterexamples. This result confirms Ball's and Ball, Lubienski, and Mewborn's arguments that majoring in mathematics does not guarantee that teachers will be equipped with sufficient subject matter knowledge for their teaching.

As prospective mathematics teachers are expected to develop fluency with proof and counterexamples in undergraduate courses for their future teaching with respect to current reform recommendations of proof and reasoning, mathematics and mathematics education professors must consider how to best communicate with each other to better connect their courses and engage students in developing knowledge and fostering understandings of proof and counterexamples.

In order to better prepare prospective mathematics teachers with content knowledge in the area of proofs and counterexamples, further research is needed, which could involve observing students' college mathematics courses, examining students' homework and examinations, and designing more mathematical statements and conducting intensive interviews with students to understand their perspectives.

In doing so, we have a better understanding of how to enhance prospective mathematics teachers' knowledge in the area of proof and counterexamples. This paper highlights the need for attention to empowering instructors in their teaching and prospective mathematics teachers in their learning to produce complete proofs and counterexamples in undergraduate mathematics courses.

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A SURVEY STUDY ON THE PATENT STATISTICS AS CONSUMER'S BEHAVIOURS INDICATORS

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ABSTRACT

This survey reviews the growing use of patent data in economic analysis. After describing some of the main characteristics of patents and patent data, it focuses on the use of patents as an indicator of technological change. Cross-sectional and time-series studies of the relationship of patents to R&D expenditures are reviewed, as well as scattered estimates of the distribution of patent values and the value of patent rights, the latter being based on recent analyses of European patent renewal data. Time-series trends of patents granted in the U.S. are examined and their decline in the 1970s is found to be an artifact of the budget stringencies at the Patent Office. The longer run downward trend in patents per R&D dollar is interpreted not as an indication of diminishing returns but rather as a reflection of the changing meaning of such data

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STUDY OF ECCENTRICITY-BASED TOPOLOGICAL INDICES OF LOCAL ANESTHETIC DRUGS

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ABSTRACT

A molecular graph is a simple graph related to the structure of a chemical compound. Let G be a simple and connected graph with the vertex set V(G) and edge set E(G). In a connected graph G, ∈ (u)is the eccentricity of vertex v , is the distance between v and a vertex farthest from v in G. Local anesthetic drugs consists of 3-parts: Lipophilic group (-Co-) or amide (-HNC-) linkage and hydrophilic group. In this paper the eccentricity-based topological indices of local anesthetic drugs are investigated.

Keywords: Eccentricity, local anesthetic drugs, QSPR, topological indices.

1. INTRODUCTION

A graph is a mathematical object that consists of set of vertices or nodes, and a set of edges that connect these nodes. A molecular graph is a simple graph such that its vertices correspond to atoms and the edges to the bonds. The simplest topological indices do not recognize double bonds and atom types and ignore hydrogen atoms and defined for connected undirected molecular graphs only [1]. The basic assumption is that different molecular structures have different chemical properties and similar molecular structures have similar molecular properties. A topological index is a numerical parameter mathematically derived from the graph structure [2]. A topological index is actually a numerical quantity associated with chemical constitution purporting for correlation of chemical structure with many physicochemical properties, chemical reactivity or you can say that biological activity [3]. In the medicine computation model, the structure of a drug is expressed as graph, where each vertex expresses an atom and each edge represents a chemical bond between these atoms [4].Topological indices are abundantly being used in the QSPR and QSAR researches. The topological indices in drugs are studied by [5-11]. A local anesthetic (LA) is a medication that causes reversible absence of pain sensation. Local anesthetics can be used as individual medicine or as a component of many combination medications [12]. Drug is any substance presented for treating, curing or preventing disease in human beings or in animals. Local anesthetics may have 3-types of systemic effects, namely: central nervous, vegetative and cardiovascular [13].Local anesthetics are drugs which upon topical application or local injection cause reversible loss of sensory perception especially of pain in a localized area of the body. The basic chemical structure of a local anesthetic molecule consists of 3 parts:

1. Lipophilic group, 2. Intermediate bond and 3. Hydrophilic group.

Let G be a graph with vertex set V(G) and edge set E(G). Let deg (u) denotes the degree of the vertex u in G. The eccentricity of a vertex u in V(G), denoted by ecc(u), and is defined as:

ecc(u) = max{d(u, v) v ∈ V(G)}. The maximum eccentricity over all vertices of G is called the diameter of G and denoted by D(G). The eccentricity–based topological indices are studied for molecular graphs by [14-19]. The multiplicative version of fifth atom bond connectivity index $ABC_5(G)$, the second multiplicative Zagreb index, $\square^*_2(G)$, fifth atom bond connectivity index $ABC_5(G)$, eccentricity-based Geometrical-arithmetic index $GA_4(G)$, Harmonic Eccentric index HEI(G), fourth Zagreb index $Zg_4(G)$, average eccentricity index aver(G),third multiplicative Zagreb index $\square^*_3(G)$,and sixth Zagreb index $Zg_6(G)$ are defined as [20-21]:

1) Fifth multiplicative atom bond connectivity index $ABC_5(G) = \prod_{uv\epsilon(G)} \sqrt{\frac{(\epsilon(u)+\epsilon(v))-2}{(\epsilon(u)\epsilon(v))}}$

2) Second multiplicative Zagreb index $\prod_2^*(G) = \prod_{uv\in E(G)}(\in(u)\in(v))$

3) Fifth atom bond connectivity index $ABC_5(G) = \sum_{uv\in(G)} \sqrt{\frac{\epsilon(u)+\epsilon(v)-2}{\epsilon(u)\epsilon(v)}}$

4) Eccentricity-based Geometrical-arithmetic index $GA_4(G) = \sum_{u\,v\,\in E(G)} 2\,\frac{\sqrt{\epsilon(u)\epsilon(v)}}{\epsilon(u)+\epsilon(v)}$

5) Harmonic Eccentric index $HEI(G) = \sum_{u\,v\,\in E(G)} \frac{2}{\epsilon(u)+\epsilon(v)}$

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6) Fourth Zagreb index $Zg_4 = \sum_{uv \in E(G)} (\in(u) + \in(v))$

7) Average eccentricity index aveg. (G) = $1/n \sum_{v \in V(G)} \in(V)$

8) Third multiplicative Zagreb index $\Pi^*_3(G) = \prod_{uv \in E(G)} (\in(u) + \in(v))$

9) Sixth Zagreb index $Zg_6 = \sum_{uv \in E(G)} (\in(u) \in(v))$.

Where $\in(u)$ is eccentricity of vertex u. The notations used in this book are standard and mainly taken from [22-28]. In this paper topological indices of multiplicative version of fifth atom bond connectivity index $ABC_5(G)$, second multiplicative Zagreb index Π^*_2 (G) , fifth atom bond connectivity index, $ABC_5(G)$, eccentricity-based Geometrical-arithmetic index $GA_4(G)$, Harmonic Eccentric index HEI(G), fourth Zagreb index $Zg_4(G)$,average eccentricity average index aver(G),third multiplicative Zagreb index $\Pi^*_3(G)$, sixth Zagreb index $Zg_6(G)$ are investigated for local anesthetic drugs.

2. MATERIALS AND METHOD

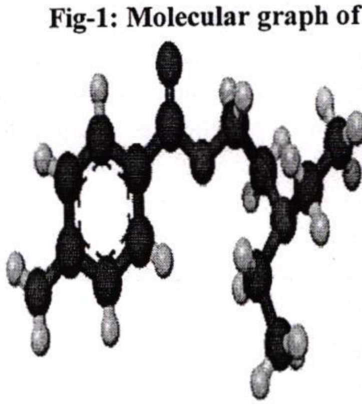
In the medicine computation model, the structure of a drug is expressed as a graph, where each vertex expresses an atom and each edge represents a chemical bond between these atoms.

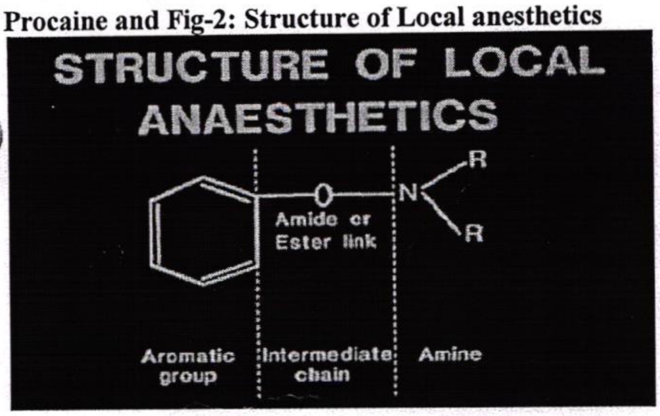
Let G be a graph with vertex set V(G) and edge set E(G). Let deg(u) denote the degree of the vertex u in G. The eccentricity of a vertex u in V(G), denoted by ecc(u), is defined as

ecc(u) = max{d(u, v) v ∈ V(G)}.

The total number of edges and vertices in each molecular graph of local anesthetic drugs are counted. In the case of degree-based topological indices the total numbers of edges are paired on the basis of degree d_u and d_v of vertices, considering this fact the edges are divided into eccentricity-based pairing. The eccentricities of 15 local anesthetic drugs are computed from their molecular graphs. The molecular graph of procaine and structure of local anesthetics are shown in figures (1) and (2).

Fig-1: Molecular graph of Procaine and Fig-2: Structure of Local anesthetics





3. RESULTS AND DISCUSSION

In this section the eccentricity-based topological indices multiplicative version of fifth atom bond connectivity index $ABC_5(G)$, second multiplicative Zagreb index $\Pi^*_2(G)$, fifth atom bond connectivity index $ABC_5(G)$, eccentricity-based Geometrical-arithmetic index $GA_4(G)$, Harmonic Eccentric index HEI(G), fourth Zagreb index $Zg_4(G)$,average eccentricity index aver(G),third multiplicative Zagreb index $\Pi^*_3(G)$, sixth Zagreb index $Zg_6(G)$ are computed for local anesthetic drugs. The Graph structure of Procaine with eccentricity values of each vertex and graph structure of Ropivacaine,Bupivaine and Levobuvaine are shown in figures (3) and (4). By means of structure analysis of local anesthetic drugs the partitions of edge set are:

E_{22}: du = dv = 2; E_{33}; du =dv = 3; E_{23}; du =2 and dv = 3; E_{12}: du =1 and dv = 2;

E_{13}: du =1 and dv = 3 .The eccentricity of each vertex is counted on from each respective graph of local anesthetic drug.

The Harmonic eccentric index for procaine is computed as: Eccentricity-edges in procaine (figure 2) are [11,12]*4, [7,8]*1, [8,9]*1, [10,11]*5, [12,7]*2,[7,6]*3 and [9,10]*1.

$$HEI(G) = \sum_{uv \in E(G)} \frac{2}{\in(u) + \in(v)}$$

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$$=\frac{2x2}{12+7}+\frac{3x2}{7+6}+\frac{1x2}{9+10}+\frac{2x4}{11+12}+\frac{2x1}{7+8}+\frac{2x1}{8+9}+\frac{2x5}{10+11}.$$

$$=\frac{4}{19}+\frac{6}{13}+\frac{2}{19}+\frac{8}{23}+\frac{2}{15}+\frac{2}{17}+\frac{10}{21}.$$

=1.056

The eccentricity-based topological indices: Harmonic eccentric index, π_3 *(G) and π_2 *(G) are computed by considering eccentricity-edges of molecular graphs of local anesthetic drugs and are given in table 1.

By using the same methodology the eccentricity-based topological indices (GA_4, Zg_6, and Average index, ABC_5, multiplicative ABC_5 and Zg_4 for local anesthetic drugs are computed and represented graphically in figure (4). Zg_4 topological index has higher values among these indices.

Fig-2: Graph structure of Procaine with eccentricity values and Fig.3. Graph structure of Ropivacaine, Bupivaine and Levobuvaine

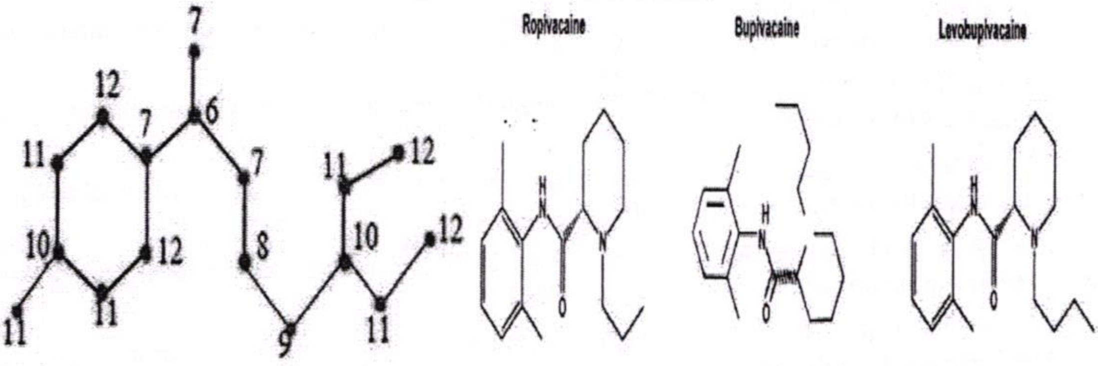
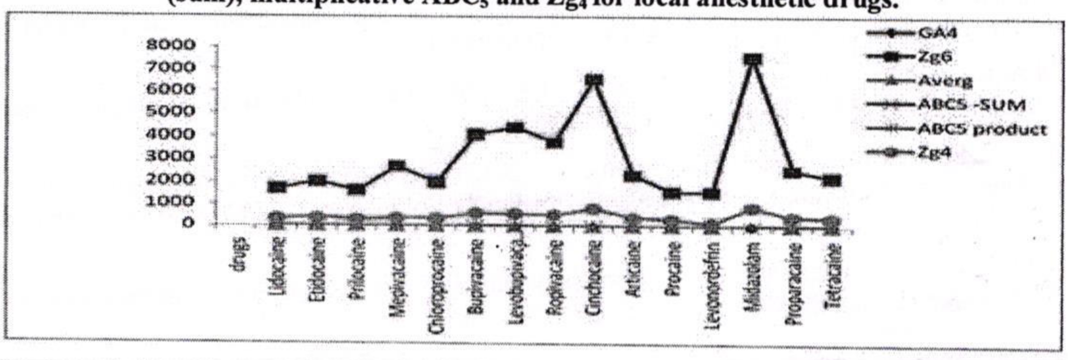


Table-1: Harmonic eccentric index, π_3 *(G) and π_2 *(G) topological indices for local anesthetic drugs.

S. N.	Local anesthetic drugs	Harmonic eccentric index	π_3 *(G)	π_2 *(G)
1	Lidocaine	1.659	$2.695*10^{10}$	$2.004*10^{14}$
2	Etidocaine	1.909	$5.603*10^{12}$	$1.218*10^{8}$
3	Prilocaine	1.639	$2.241*10^{12}$	$4.872*10^{17}$
4	Mepivacaine	1.667	$2.027*10^{13}$	$2.026*10^{19}$
5	Chloroprocaine	1.775	$3.362*10^{13}$	$4.228*10^{19}$
6	Bupivacaine	1.523	$2.914*10^{20}$	$3.358*10^{30}$
7	Levobupivacaine	1.618	$3.706*10^{8}$	$3.456*10^{27}$
8	Ropivacaine	1.644	$6.972*10^{16}$	$4.725*10^{24}$
9	Cinchocaine	1.643	$9.714*10^{16}$	$2.022*10^{25}$
10	Articaine	1.751	$1.455*10^{14}$	$3.909*10^{20}$
11	Procaine	1.056	$6.936*10^{10}$	$3.161*10^{15}$
12	Levonordefrin	1.925	$1.394*10^{6}$	$1.412*10^{8}$
13	Midazolam	1.535	$5.103*10^{19}$	$8.901*10^{29}$
14	Proparacaine	1.963	$2.161x10^{13}$	$1.282*10^{19}$
15	Tetracaine	1.792	$5.763*10^{11}$	$6.771*10^{16}$

Fig-4: Graphical representation of eccentricity-based topological indices (GA_4, Zg_6, Average index, ABC_5 (sum), multiplicative ABC_5 and Zg_4 for local anesthetic drugs.



4. CONCLUSION

The eccentricity-based HEI(G) has been computed for local anesthetic drugs .The eccentricity-based topological indices: multiplicative version of fifth atom bond connectivity index $ABC_5(G)$, second multiplicative Zagreb index Π^*_2 (G) , fifth atom bond connectivity index, $ABC_5(G)$, eccentricity-based Geometrical-arithmetic index $GA_4(G)$, fourth Zagreb index $Zg_4(G)$,average eccentricity average index aver(G), third multiplicative Zagreb index $\Pi^*_3(G)$, sixth Zagreb index $Zg_6(G)$ are studied for local anesthetic drugs. Local anesthetics can be used as individual medicine or as a component of many combinations medications. The eccentricity-based topological indices are used in QSAR/QSPR studies and design of new drugs.

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MATHEMATICAL MODELING OF PERFORMANCE BASED ANALYSIS OF AN INTEGRATED CELLULAR AND AD HOC RELAY SYSTEM

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ABSTRACT

In this paper, we propose a new wireless system architecture based on the integration of cellular and modern ad hoc relaying technologies. It can efficiently balance traffic loads and share channel resource between cells by using ad hoc relaying stations to relay traffic from one cell to another dynamically. However, the application demand and allocation could lead to congestion, if the network has to maintain such high resources for quality of service (QoS) requirements of the applications.

In our system, handoff area and queue are taken into consideration and new and handoff calls are given priority, respectively. We analyze the system performance in terms of the call blocking probability and queueing delay for new call requests and call dropping probability for handoff requests. Numerical illustrations are provided with the help of Successive Overrelaxation Method (SOR). In order to improve the performance of base station, the trade off between number of services channel and QoS of base station must be considered.

Keywords: **Ad Hoc networks, Cellular architecture, Relaying, Markovian model, Integration, Blocking probability, Queueing system modeling**

1. Introduction

Mobile communications have achieved rapid growth in recent years and the further advancement is expected to realize the future ubiquitous society. However, since the bandwidth is limited, it is very important to consider how to use the limited resources efficiently. Recently, the demand for wireless communications has grown tremendously and a lot of fundamental challenges and issues on wireless networks and mobile computing have been identified such as handover and call admission, fixed and dynamic channel assignment, data management, routing in wireless ad hoc networks, etc. As the demand of seamless communications is growing and the number of wireless users is increasing, effective call handling is becoming more and more important to utilize scarce radio resources more efficiently. In order to support dynamically arriving and departing calls effectively, size and hierarchy of cells, partitioned areas to handle wireless terminals, have to be carefully designed to maximize coverage area and to support user calls.

Ad hoc network systems are self organized, self managing, flexible and the multi hop communication in ad hoc networks leads to extending the coverage of existing wireless access technologies. It can be represented the improvement of reducing the cost of wireless access infrastructure. With the purpose of reusing the limited radio resources and reducing power consumption, a cellular networks require fixed base stations that are interconnected by a wired backbone and base stations are very important for the networks. Wu and Chuang (2001) studies dynamic QoS allocation for multimedia ad hoc wireless networks. Wu et al. (2003) analyzed a survey of mobile IP in cellular and mobile ad hoc network environments.[5] Remondo and Niemegeers (2003) gave ad hoc networking in future wireless communications. Zheng et al. (2004) performed recent advances in mobility modeling for mobile ad hoc networks. Ad hoc networks have multi-hop communication function and if the distance between two mobile stations is large, they can communicate with each other to relay stations. Murillo-Perez et al. (2009) studied the impact of mobility on OFDMA-based cellular systems with reuse partitioning. Sharma and Jain (2010) analyzed multihop cellular networks: a review. Pandey et al. (2013) studied optimum relay selection for energy efficient adhoc networks. Development of mobile adhoc network based on QOS routing protocol for healthcare was suggested by Hussain (2015). Guizani (2016) gave relay attacks concerns in wireless adhoc and sensor networks.

Cellular network system consists of two types of calls new calls and handover calls. The handoff calls are those calls, which are already ongoing and move into a new cell and need to connect to a new base station. The blocking probability of the hand off calls is an important demand direct, reliable and efficient connection. As the system, the study on the system with guard channels was represented by Guerin (1998). In this system, some number of channels is used exclusively for handoff calls because blocking of a call in progress is less desirable than the blocking of a new call. Zhang et al. (2003) have done approximation approach on performance evaluation for guard channel scheme. Yavuz and Leung (2006) presented computationally efficient method to evaluate the performance of guard-channel-based call admission control in cellular networks. Cruz-Perez et al. (2011) proposed approximated mathematical analysis methods of guard channel based call admission control in cellular networks. Kim et al. (2012) gave an analytical approach to the analysis of guard channel based call admission control in wireless cellular networks. Saritha, and Viswanatham (2014) presented a new approach for channel reservation and allocation to improve quality of service in vehicular communications.

Moreever, the channel reservation scheme that priority is given to handoff calls reduces the total carried traffic. The analysis of the system which some channels are reserved exclusively for handoff calls and there are the queues for the new and handoff calls. Jain (2000) presented prioritized channel assignment in mixed media cellular radio system. Salamah and Lababidi (2005) proposed dynamically adaptive channel reservation scheme for cellular network. Xhafa and Tonguz (2008) gave handover performance of priority schemes in cellular networks. Soh and Kim (2009) presented a predictive bandwidth reservation scheme using mobile positioning and road topology information. Sharma and Purohit (2011) suggested prioritized channel assignment to multimedia calls in wireless cellular networks.

Integrated cellular and ad hoc systems were represented in Wu et al. (2001). However, this paper focus on the coverage of a relay station and handoff area which is unstable, complicated and important is not taken into consideration. Fang and Chlamtac (2002) proposed analytical generalized results for handoff probability in wireless networks. Dharmraja et al. (2003) studied on modeling of wireless networks with general distributed handoff interarrival times. Wu et al. (2005) gave handoff performance of the integrated cellular and adhoc relaying (iCAR) system. Modeling of the system with handoff area represents the situation of a boundary in cells more than that of the system without handoff area. Bhattachary et al. (2008) gave traffic model and performance analysis of cellular mobile systems for general distributed handoff traffic and channel allocation. Kumar and Tripathi (2009) studied adaption of the preemptive handoff scheme in an integrated mobile communication environment. Halgamuge et al. (2011) performed handoff optimization using hidden markov model. Maximisation of correct handover probability and data throughput in vehicular networks was applied by Banda and Mzyece (2014).

It is not always possible to find analytic solution, as such numerical approaches thus appe to be the only way to obtain results. Garcia et al. (2005) studied admission control policies in the multi service cellular networks. Blocked handover sessions are queued up but an exponential deadline is defined beyond which a session is forced to terminate. We obtain on the numerical solution of the steady state Kolmogorov equations of the continuous time markov chain describing the system dynamics. Thus, the approximate model presented in this paper can be successfully used for an accurate performance evaluation, design and planning of cellular mobile telephone networks.

In this paper, we consider the integrated ad hoc cellular network system where handoff area is taken into consideration and new and handoff calls are given priority. The rest of the paper is structured as follows. In section 2, Markovian model is described by stating the requisite assumptions and notations being used in the formulation of the mathematical model. The governing steady state equations are presented in section 4. Various performance measures are established in section 5. The steady state probability vectors are obtained by using successive overrelaxation method in the next section 6. Sensitivity analysis by taking numerical illustration is carried out by varying different parameters in section 7. Finally, we wind up our study by giving concluding remarks in section 8.

2. System Model

In this paper, there are three cells, cell A, cell B and cell C in this system. N channels are assigned to cell A, cell B and cell C, respectively. Handoff area is defined as the overlap region of cells and handoff calls are the calls that move to the neighbouring cell and handoff process is done within the handoff area. We consider the integrated ad hoc cellular network system, new calls are given the priority of relaying and new calls in handoff area and handoff calls are given the priority of queueing. When the relay station is set in handoff area and the area covered by the relay station is represented by the ratio l.

If there are no channels available in cell A, cell B and cell C on arrival, the new calls are covered by a relay station can be relayed to the other cell. New calls in handoff area and handoff calls use a channel for cell A or cell B or cell C with the probability of 0.3. New calls in handoff area and handoff calls can wait in a queue with capacity Q while they are in handoff area if there are no channels are available in cells A, B, C and they can not use the channels in the other cell. New calls in handoff area and handoff calls are blocked if the queue is full. If the calls in the queue can not get a new channel while they are in handoff area, they leave the queue halfway.

For the development of traffic model, we made the following assumptions:

- There are N channels allocated in a each cell A, cell B and cell C, respectively
- The arrival pattern of new calls and handoff calls follow the Poisson process.
- The call holding times of new and handoff calls are exponential distributed.
- The time between two successive handover requests of a call is a random variable and follows the exponential distribution.

We shall use the following notations for mathematical formulation purpose:

N Total number of channels.

I Number of being used channels of cell A

J Number of being used channels of cell B

K Number of being used channels of cell C

S Number of calls in the queue

$\square_{nA}$ Arrival rate of new calls in cell A.

$\square_{nB}$ *Arrival rate of new calls in cell B.*

$\square_{nC}$ Arrival rate of new calls in cell C.

$\square_{hh}$ The arrival rate of handoff calls.

$\square_{\square h}$ The arrival rate of handoff area.

$\square_{h}$ The arrival rate of new calls in handoff area and handoff calls.

$\square$ Call holding times of new calls

$\square_{d}$ Call holding times of handoff calls

$\square_{w}$ Dwell times of the cells and handoff area.

$\square_{\square d}$ Service rate of releasing a channel.

$\square_{tw}$ Service rate of leaving a queue halfway.

$P_{i,j,k,s}$ The steady state probabilities of the cells.

Q The capacity of queue.

l The ratio of relay station of the cells with the probability 0.3.

B_{nA} Blocking probability of new calls in cell A.

B_{nB} Blocking probability of new calls in cell B.

B_{nC} Blocking probability of new calls in cell C.

B_{h} Blocking probability of handoff calls.

T_{ch} Total carried traffic.

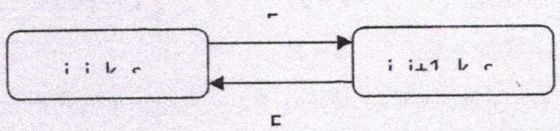
Define: $\square_{h} = \square_{\square h} + \square_{hh}$, $\square_{td} = \square + \square_{d}$ and $\square_{tw} = \square + \square_{w}$

3. Queueing Network Model

We define the state of the system as (i, j, k, s) where i denotes the number of being used channels of cell A, j does the number of being used channels of cell B, k does the number of being used channels of cell C and s does the number of call in the queue. M channels are assigned to cell A, cell B and cell C. The state dependent arrival rates and service rates for various states are shown in table 1, respectively.

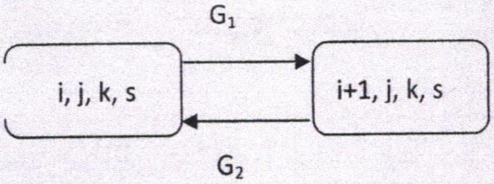
The state dependent arrival rate and channel holding time is defined as

F:

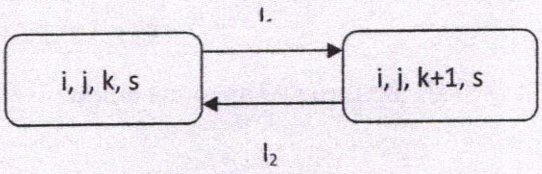


Condition	Arrival States	Service States
If i > j	$F_1=\square_{nB}+\square_h$	$F_2=(j+1)\ \square_{td}$
If i = j	$F_1=\square_{nB}+l\ \square_h$	$F_2=(j+1)\ \square_{td}$
If i < j	$F_1=\square_{nB}$	$F_2=(j+1)\ \square_{td}$
If i=M	$F_1=\square_{nB}+\square_h+l\ \square_{nA}$	$F_2=(j+1)\ \square_{td}$

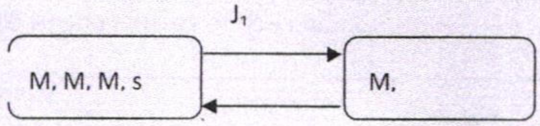
G:



Condition	Arrival States	Service States
If j < i	$G_1=\square_{nA}$	$G_2=(i+1)\ \square_{td}$
If j = i	$G_1=\square_{nA}+l\ \square_h$	$G_2=(i+1)\ \square_{td}$
If j > i	$G_1=\square_{nA}+\square_h$	$G_2=(i+1)\ \square_{td}$
If j=M	$G_1=\square_{nA}+\square_h+l\ \square_{nB}$	$G_2=(i+1)\ \square_{td}$



Condition	Arrival States	Service States
i, j > k, if $1 \le k \le M-1$,	I_1=⍰$_{nC}$ + ⍰$_{h}$, i > k I_1=⍰$_{nC}$ j > k	I_2= (k+1) ⍰$_{td}$
i = j=k, if $1 \le k \le M-1$,	I_1=⍰$_{nC}$ + *l* ⍰$_{h}$	I_2= (k+1) ⍰$_{td}$
i, j < k, if $1 \le k \le M-1$	I_1=⍰$_{nC}$, i < k I_1=⍰$_{nC}$ + ⍰$_{h}$ j < k	I_2= (k+1) ⍰$_{td}$
If i=k=M	I_1=⍰$_{nB}$ + ⍰$_{h}$+*l* ⍰$_{nA}$+ *l* ⍰$_{nC}$	I_2= (j+1) ⍰$_{td}$
If j=k=M	I_1=⍰$_{nA}$ + ⍰$_{h}$+*l* ⍰$_{nB}$+ *l* ⍰$_{nC}$	I_2= (i+1) ⍰$_{td}$
If i=j=M	I_1=⍰$_{nc}$ + ⍰$_{h}$+*l* ⍰$_{nA}$+ *l* ⍰$_{nB}$	I_2= (k+1) ⍰$_{td}$



J:

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Table 1: Transition flow rates

4. Governing Equations

Using state transition rates as given in table 1 (A), for the computational purpose, the state governing

equations for the fixed parameter M = 3 , Q=3 are constructed as given below:

$$-[(\lambda_{nA}+l\,\lambda_h)+(\lambda_{nB}+l\,\lambda_h)+(\lambda_{nC}+l\,\lambda_h)]P_{0,0,0,0}+\mu_{td}P_{1,0,0,0}+\mu_{td}P_{0,0,1,0}+\mu_{td}\,P_{0,1,0,0}=0 \quad \text{... (1)}$$

$$-[\lambda_{nB}+\lambda_{nC}+\mu_{td}+(\lambda_{nA}+\lambda_h)]P_{0,1,0,0}+\mu_{td}P_{0,1,1,0}+\mu_{td}P_{1,1,0,0}+2\mu_{td}\,P_{0,2,0,0}+(\lambda_{nB}+l\,\lambda_h)P_{0,0,0,0}=0 \quad \text{... (2)}$$

$$-[\lambda_{nB}+\lambda_{nC}+2\mu_{td}+(\lambda_{nA}+\lambda_h)]P_{0,2,0,0}+\mu_{td}P_{0,2,1,0}+\mu_{td}P_{1,2,0,0}+3\mu_{td}\,P_{0,3,0,0}+(\lambda_{nB})P_{0,1,0,0}=0 \quad \text{... (3)}$$

$$-[(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+\lambda_{nC})+(l\,\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+\lambda_{nC})+3\mu_{td}]P_{0,3,0,0}+\mu_{td}P_{1,3,0,0}$$
$$+\mu_{td}P_{0,3,1,0}+(\lambda_{nB})P_{0,2,0,0}=0 \quad \text{... (4)}$$

$$-[\lambda_{nB}+(\lambda_{nA}+l\,\lambda_h)+\mu_{td}+(\lambda_{nC}+\lambda_h)]P_{0,0,1,0}+\mu_{td}P_{0,1,1,0}+\mu_{td}P_{1,0,1,0}+2\mu_{td}\,P_{0,0,2,0}$$
$$+(\lambda_{nC}+l\,\lambda_h)P_{0,0,0,0}=0 \quad \text{... (5)}$$

$$-[(\lambda_{nA}+\lambda_h)+2\mu_{td}+(\lambda_{nB}+l\,\lambda_h)+\lambda_{nC}]P_{0,1,1,0}+\mu_{td}P_{1,1,1,0}+2\mu_{td}P_{0,1,2,0}+2\mu_{td}\,P_{0,2,1,0}$$
$$+(\lambda_{nC}+\lambda_h)P_{0,0,1,0}+\lambda_{nC}P_{0,1,0,0}=0 \quad \text{... (6)}$$

$$-[(\lambda_{nB})+2\mu_{td}+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+\lambda_{nC}]P_{0,1,2,0}+\mu_{td}P_{1,1,2,0}+2\mu_{td}P_{0,2,2,0}+3\mu_{td}\,P_{0,1,3,0}$$
$$+(\lambda_{nB}+l\,\lambda_h)P_{0,0,2,0}+\lambda_{nC}P_{0,1,1,0}=0 \quad \text{... (7)}$$

$$-[(\lambda_{nB})+3\mu_{td}+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)]P_{0,1,3,0}+\mu_{td}P_{1,1,3,0}+2\mu_{td}P_{0,2,3,0}+(\lambda_{nB}+l\,\lambda_h)P_{0,0,3,0}$$
$$+\lambda_{nC}P_{0,1,2,0}=0 \quad \text{... (8)}$$

$$-[(\lambda_{nB})+3\mu_{td}+(\lambda_{nA}+\lambda_h)+\lambda_{nC}]P_{0,2,1,0}+\mu_{td}P_{1,2,1,0}+2\mu_{td}P_{0,2,2,0}+3\mu_{td}\,P_{0,3,1,0}$$
$$+(\lambda_{nB})P_{0,1,1,0}+\lambda_{nC}P_{0,2,0,0}=0 \quad \text{... (9)}$$

$$-[(\lambda_{nB}+l\,\lambda_h)+3\mu_{td}+(\lambda_{nA}+\lambda_h)+\lambda_{nC}]P_{0,2,2,0}+\mu_{td}P_{1,2,2,0}+3\mu_{td}P_{0,3,2,0}+3\mu_{td}\,P_{0,2,3,0}$$
$$+(\lambda_{nB})P_{0,1,2,0}+\lambda_{nC}P_{0,2,1,0}=0 \quad \text{... (10)}$$

$$-[(\lambda_{nB}+\lambda_h)+2\mu_{td}+3\mu_{td}+(\lambda_{nA}+\lambda_h)]P_{0,2,3,0}+\mu_{td}P_{1,2,3,0}+3\mu_{td}P_{0,3,3,0}$$
$$+(\lambda_{nB})P_{0,1,3,0}+(\lambda_{nC}+l\,\lambda_h)P_{0,2,2,0}=0 \quad \text{... (11)}$$

$$-[(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})+\mu_{td}+3\mu_{td}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_h+\lambda_{nC})]P_{0,3,1,0}+\mu_{td}P_{1,3,1,0}$$
$$+2\mu_{td}P_{0,3,2,0}+(\lambda_{nB})P_{0,2,1,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{0,3,0,0}=0 \quad \dots (12)$$

$$-[(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)+2\mu_{td}+3\mu_{td}+(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\lambda_{nC})]P_{0,3,2,0}+\mu_{td}P_{1,3,2,0}$$
$$+2\mu_{td}P_{0,3,2,0}+(\lambda_{nB}+l\,\lambda_h)P_{0,2,2,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{0,3,1,0}=0 \quad \dots (13)$$

$$-[(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})+3\mu_{td}+3\mu_{td}]P_{0,3,3,0}+\mu_{td}P_{1,3,3,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{0,2,3,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{0,3,2,0}=0 \quad \dots (14)$$

$$-[\lambda_{nA}+(\lambda_{nC}+l\,\lambda_h)+\mu_{td}+(\lambda_{nB}+\lambda_h)]P_{1,0,0,0}+\mu_{td}P_{1,1,0,0}+\mu_{td}P_{1,0,1,0}+2\mu_{td}P_{2,0,0,0}$$
$$+(\lambda_{nA}+l\,\lambda_h)P_{0,0,0,0}=0 \quad \dots (15)$$

$$-[(\lambda_{nA}+l\,\lambda_h)+2\mu_{td}+(\lambda_{nB}+l\,\lambda_h)+\lambda_{nC}]P_{1,1,0,0}+\mu_{td}P_{1,1,1,0}+2\mu_{td}P_{1,2,0,0}+2\mu_{td}P_{2,1,0,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{1,0,0,0}+(\lambda_{nA}+\lambda_h)P_{0,1,0,0}=0 \quad \dots (16)$$

$$-[(\lambda_{nB})+2\mu_{td}+2\mu_{td}+(\lambda_{nA}+\lambda_h)+\lambda_{nC}]P_{1,2,0,0}+\mu_{td}P_{1,2,1,0}+2\mu_{td}P_{2,2,0,0}+3\mu_{td}P_{1,3,0,0}$$
$$+(\lambda_{nB}+l\,\lambda_h)P_{1,1,0,0}+(\lambda_{nA}+\lambda_h)P_{0,2,0,0}=0 \quad \dots (17)$$

$$-[(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})+\mu_{td}+3\mu_{td}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)]P_{1,3,0,0}+2\mu_{td}P_{2,3,0,0}$$
$$+\mu_{td}P_{1,3,1,0}+(\lambda_{nB})P_{1,2,0,0}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda_h)P_{0,3,0,0}=0 \quad \dots (18)$$

$$-[(\lambda_{nB})+2\mu_{td}+(\lambda_{nC}+l\,\lambda_h)+\lambda_{nA}]P_{1,0,1,0}+\mu_{td}P_{1,1,1,0}+2\mu_{td}P_{2,0,1,0}+2\mu_{td}P_{1,0,2,0}$$
$$+(\lambda_{nC}+l\,\lambda_h)P_{1,0,0,0}+\lambda_{nA}P_{0,0,1,0}=0 \quad \dots (19)$$

$$-[(\lambda_{nB}+\lambda_h)+3\mu_{td}+(\lambda_{nC})+2\mu_{td}]P_{1,0,2,0}+\mu_{td}P_{1,1,2,0}+3\mu_{td}P_{1,0,3,0}+\lambda_{nA}P_{1,0,2,0}$$
$$+(\lambda_{nA}+l\,\lambda_h)P_{0,0,2,0}+\lambda_{nc}P_{1,0,1,0}=0 \quad \dots (20)$$

$$-[(\lambda_{nB}+\lambda_h)+3\mu_{td}+(\lambda_{nA})+\mu_{td}]P_{1,0,3,0}+\mu_{td}P_{1,1,3,0}+2\mu_{td}P_{2,0,3,0}+\lambda_{nC}P_{1,0,2,0}$$
$$+(\lambda_{nA}+l\,\lambda_h)P_{0,0,3,0}=0 \quad \dots (21)$$

$$-[(\lambda_{nB}+l\,\lambda_h)+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+\mu_{td}+(\lambda_{nC}+l\,\lambda_h)]P_{1,1,1,0}+2\mu_{td}P_{1,2,1,0}+2\mu_{td}P_{2,1,1,0}$$
$$+\lambda_{nC}\,P_{1,1,0,0}+(\lambda_{nA}+\lambda_h)P_{0,1,1,0}+\lambda_{nB}\,P_{1,0,1,,0}=0 \quad \text{... (22)}$$

$$-[(\lambda_{nB})+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+\mu_{td}+(\lambda_{nC}+\lambda_h)]P_{1,1,2,0}+2\mu_{td}P_{1,2,2,0}+2\mu_{td}P_{2,1,2,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{1,0,2,0}+(\lambda_{nA}+\lambda_h)P_{0,1,2,0}+(\lambda_{nC}+l\,\lambda_h)P_{1,1,1,,0}+3\mu_{td}P_{1,1,3,0}=0 \quad \text{... (23)}$$

$$-[(\lambda_{nB})+\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+\mu_{td}]P_{1,1,3,0}+2\mu_{td}P_{1,2,3,0}+2\mu_{td}P_{2,1,3,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{1,0,3,0}+(\lambda_{nA}+\lambda_h)P_{0,1,3,0}=0 \quad \text{... (24)}$$

$$-[(\lambda_{nB})+2\mu_{td}+(\lambda_{nA}+\lambda_h)+\mu_{td}+(\lambda_{nC})+\mu_{td}]P_{1,2,1,0}+3\mu_{td}P_{1,3,1,0}+2\mu_{td}P_{2,2,1,0}$$
$$+\lambda_{nC}\,P_{1,2,0,0}+(\lambda_{nB}+l\,\lambda_h)P_{1,1,1,0}+(\lambda_{nA}+\lambda_h)P_{0,2,1,,0}+2\mu_{td}P_{1,2,2,0}=0 \quad \text{... (25)}$$

$$-[(\lambda_{nB})+2\mu_{td}+(\lambda_{nA}+\lambda_h)+\mu_{td}+(\lambda_{nC}+l\,\lambda_h)+\mu_{td}+2\mu_{td}]P_{1,2,2,0}+3\mu_{td}P_{1,2,3,0}+2\mu_{td}P_{2,2,2,0}$$
$$+\lambda_{nB}\,P_{1,1,2,0}+\lambda_{nC}\,P_{1,2,1,0}+(\lambda_{nB}+l\,\lambda_h)P_{1,1,1,0}+(\lambda_{nA}+\lambda_h)P_{0,2,2,,0}+2\mu_{td}P_{1,3,2,0}=0 \quad \text{... (26)}$$

$$-[(\lambda_{nB})+2\mu_{td}+(\lambda_{nA}+\lambda_h)+\mu_{td}+3\mu_{td}]P_{1,2,3,0}+3\mu_{td}P_{1,3,3,0}+2\mu_{td}P_{2,2,3,0}$$
$$+\lambda_{nB}\,P_{1,1,3,0}+(\lambda_{nC}+l\,\lambda_h)P_{1,2,2,0}+(\lambda_{nA}+\lambda_h)P_{0,2,3,0}=0 \quad \text{... (27)}$$

$$-[(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)+\mu_{td}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda_h)+3\mu_{td}+\mu_{td}]P_{1,3,1,0}+2\mu_{td}P_{1,3,2,0}$$
$$+2\mu_{td}P_{2,3,1,0}+\lambda_{nB}\,P_{1,2,1,0}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda)P_{0,3,1,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{0,2,1,,0}=0 \quad \text{... (28)}$$

$$-[(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)+\mu_{td}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda_h)+3\mu_{td}+2\mu_{td}]P_{1,3,2,0}+2\mu_{td}P_{2,3,2,0}$$
$$+3\mu_{td}P_{1,3,3,0}+(\lambda_{nB}+l\,\lambda_h)P_{1,2,2,0}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda)P_{0,3,2,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{1,3,1,,0}=0 \quad \text{... (29)}$$

$$-[\mu_{td}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda_h)+3\mu_{td}+3\mu_{td}]P_{1,3,3,0}+2\mu_{td}P_{2,3,3,0}$$
$$+(\lambda_{nB})P_{1,2,3,0}+(\lambda_{nA}+l\,\lambda_{nB}+l\,\lambda_{nC}+\lambda)P_{0,3,3,0}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)P_{1,3,2,,0}=0 \quad \text{... (30)}$$

$$-\left[(\lambda_{nB}+\lambda_h)+2\mu_{td}+(\lambda_{nA})+(\lambda_{nC}+l\,\lambda_h)\right]P_{2,0,0,0}+\mu_{td}P_{2,0,1,0}+\mu_{td}P_{2,,1,0,0}$$
$$+\lambda_{nA}\,P_{1,0,0,0}+3\mu_{td}P_{3,0,0,0}=0 \qquad \dots (31)$$

$$-\left[(\lambda_{nB}+\lambda_h)+2\mu_{td}+\mu_{td}+(\lambda_{nA})+(\lambda_{nC})\right]P_{2,1,0,0}+2\mu_{td}P_{2,2,0,0}+\mu_{td}P_{2,1,,1,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{2,0,0,0}+(\lambda_{nA}+l\,\lambda_h)P_{1,1,0,0}+3\mu_{td}P_{3,1,0,0}=0 \qquad \dots (32)$$

$$-\left[(\lambda_{nB}+l\,\lambda_h)+2\mu_{td}+2\mu_{td}+(\lambda_{nA}+l\,\lambda_h)+(\lambda_{nC})\right]P_{2,2,0,0}+3\mu_{td}P_{2,3,0,0}+\mu_{td}P_{2,2,,1,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{2,1,0,0}+(\lambda_{nA}+\lambda_h)P_{1,2,0,0}+3\mu_{td}P_{3,2,0,0}=0 \qquad \dots (33)$$

$$-\left[(\lambda_{nA}+l\,\lambda_{nB}+l\lambda_{nC}+\lambda_h)+2\mu_{td}+3\mu_{td}+(l\,\lambda_{nA}+l\,\lambda_{nB}+\lambda_{nC}+\lambda_h)\right]P_{2,3,0,0}+3\mu_{td}P_{3,3,0,0}+\mu_{td}P_{2,3,,1,0}$$
$$+(\lambda_{nA}+l\,\lambda_{nB}+l\lambda_{nC}+\lambda_h)P_{1,3,0,0}+(\lambda_{nB})P_{2,2,0,0}=0$$

$$\dots (34)$$

$$-\left[(\lambda_{nC}+\lambda_h)+\mu_{td}+2\mu_{td}+(\lambda_{nA})+(\lambda_{nB}+\lambda_h)\right]P_{2,0,1,0}+2\mu_{td}P_{2,0,2,0}+\mu_{td}P_{2,1,,1,0}$$
$$+(\lambda_{nC}+l\,\lambda_h)P_{2,0,0,0}+(\lambda_{nA})P_{1,0,1,0}+3\mu_{td}P_{3,0,1,0}=0 \qquad \dots (35)$$

$$-\left[(\lambda_{nB}+\lambda_h)+2\mu_{td}+2\mu_{td}+(\lambda_{nA})+(\lambda_{nC})\right]P_{2,0,2,0}+3\mu_{td}P_{2,0,3,0}+\mu_{td}P_{2,1,2,0}$$
$$+(\lambda_{nA})P_{1,0,2,0}+(\lambda_{nC})P_{2,0,1,0}+3\mu_{td}P_{3,0,2,0}=0 \qquad \dots (36)$$

$$-\left[(\lambda_{nB}+\lambda_h)+2\mu_{td}+3\mu_{td}+(\lambda_{nA})\right]P_{2,0,3,0}+3\mu_{td}P_{3,0,3,0}+\mu_{td}P_{2,1,3,0}$$
$$+(\lambda_{nA})P_{1,0,3,0}+(\lambda_{nC})P_{2,0,2,0}=0 \qquad \dots (37)$$

$$-\left[(\lambda_{nB}+\lambda_h)+\mu_{td}+2\mu_{td}+(\lambda_{nA})+(\lambda_{nC}+l\,\lambda_h)+\mu_{td}\right]P_{2,1,1,0}+2\mu_{td}P_{2,2,1,0}+3\mu_{td}P_{3,1,,1,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{2,0,1,0}+(\lambda_{nA}+l\,\lambda_h)P_{1,1,1,0}+2\mu_{td}P_{2,1,2,0}+\lambda_{nC2,1,0,0}=0 \qquad \dots (38)$$

$$-\left[(\lambda_{nC}+l\,\lambda_h)+\mu_{td}+2\mu_{td}+(\lambda_{nA})+(\lambda_{nB})+2\mu_{td}\right]P_{2,1,2,0}+3\mu_{td}P_{3,1,2,0}+2\mu_{td}P_{2,1,2,0}$$
$$+(\lambda_{nA}+l\,\lambda_h)P_{2,1,2,0}+(\lambda_{nB}+\lambda_h)P_{2,0,2,0}+3\mu_{td}P_{2,1,3,0}+\lambda_{nC}P_{2,1,1,0}=0 \qquad \dots (39)$$

$$-\left[(\lambda_{nA})+\mu_{td}+2\mu_{td}+(\lambda_{nB})+3\mu_{td}\right]P_{2,1,3,0}+3\mu_{td}P_{3,1,3,0}+2\mu_{td}P_{2,2,3,0}$$
$$+(\lambda_{nA}+l\,\lambda_h)P_{1,1,3,0}+(\lambda_{nB}+\lambda_h)P_{2,0,3,0}+(\lambda_{nC}+l\,\lambda_h)P_{2,1,2,0}=0 \qquad \dots (40)$$

$$-[(\lambda_{nB}+l\lambda_h)+2\mu_{td}+2\mu_{td}+(\lambda_{nA}+l\lambda_h)+(\lambda_{nC})+\mu_{td}]P_{2,2,1,0}+3\mu_{td}P_{2,3,1,0}+3\mu_{td}P_{3,2,1,0}$$
$$+(\lambda_{nB}+\lambda_h)P_{2,1,1,0}+(\lambda_{nA}+\lambda_h)P_{1,2,1,0}+2\mu_{td}P_{2,2,2,0}+\lambda_{nC}P_{2,2,0,0}=0 \quad \ldots (41)$$

$$-[(\lambda_{nB}+l\lambda_h)+2\mu_{td}+2\mu_{td}+(\lambda_{nA}+l\lambda_h)+(\lambda_{nC}+l\lambda_h)+2\mu_{td}]P_{2,2,2,0}+3\mu_{td}P_{2,2,3,0}+3\mu_{td}P_{3,2,2,0}$$
$$+(\lambda_{nA}+\lambda_h)P_{1,2,2,0}+(\lambda_{nC}+\lambda_h)P_{2,2,1,0}+2\mu_{td}P_{2,1,2,0}+3\mu_{td}P_{2,3,2,0}=0 \quad \ldots (42)$$

$$-[(\lambda_{nB}+l\lambda_h)+2\mu_{td}+2\mu_{td}+(\lambda_{nA}+l\lambda_h)+3\mu_{td}]P_{2,2,3,0}+3\mu_{td}P_{3,2,3,0}+3\mu_{td}P_{2,3,3,0}$$
$$+(\lambda_{nA}+\lambda_h)P_{1,2,3,0}+(\lambda_{nC}+l\lambda_h)P_{2,2,2,0}+3\mu_{td}P_{2,3,2,0}=0 \quad \ldots (43)$$

$$-[(l\lambda_{nA}+l\lambda_{nB}+\lambda_{nC}+\lambda_h)+\mu_{td}+2\mu_{td}+(\lambda_{nA}+\lambda_h+l\lambda_{nB}+l\lambda_{nC})+3\mu_{td}]P_{2,3,1,0}+2\mu_{td}P_{2,3,2,0}$$
$$+(\lambda_{nA}+\lambda_h+l\lambda_{nB}+l\lambda_{nC})P_{1,3,1,0}+(\lambda_{nB})P_{2,2,1,0}+3\mu_{td}P_{3,3,1,0}+(l\lambda_{nA}+\lambda_h+l\lambda_{nB}+\lambda_{nC})P_{2,3,0,0}=0 \quad \ldots (44)$$

$$-[(l\lambda_{nA}+l\lambda_{nB}+\lambda_{nC}+\lambda_h)+2\mu_{td}+2\mu_{td}+(\lambda_{nA}+\lambda_h+l\lambda_{nB}+l\lambda_{nC})+3\mu_{td}]P_{2,3,2,0}+3\mu_{td}P_{3,3,2,0}$$
$$+(l\lambda_{nA}+\lambda_h+l\lambda_{nB}+\lambda_{nC})P_{2,3,1,0}+(\lambda_{nB}+\lambda_h)P_{2,2,2,0}+3\mu_{td}P_{2,3,3,0}+(\lambda_{nA}+\lambda_h+l\lambda_{nB}+l\lambda_{nC})P_{1,3,2,0}=0 \quad \ldots (45)$$

$$-[(l\lambda_{nA}+l\lambda_{nB}+\lambda_{nC}+\lambda_h)+3\mu_{td}+(\lambda_{nA}+\lambda_h+l\lambda_{nB}+l\lambda_{nC})+(\lambda_{nB}+\lambda_h)]P_{2,3,3,0}+3\mu_{td}P_{3,3,2,0}$$
$$+(\lambda_{nA}+\lambda_h+l\lambda_{nB}+l\lambda_{nC})P_{1,3,3,0}+(\lambda_{nB}+\lambda_h)P_{2,2,3,0}+3\mu_{td}P_{3,3,3,0}+(l\lambda_{nA}+\lambda_h+l\lambda_{nB}+\lambda_{nC})P_{2,3,2,0}=0 \quad \ldots (46)$$

$$-[(\lambda_{nB}+\lambda_h+l\lambda_{nC}+l\lambda_{nA})+3\mu_{td}+(\lambda_{nA}+\lambda_h+l\lambda_{nB}+l\lambda_{nC})]P_{3,0,0,0}+\mu_{td}P_{3,1,0,0}$$
$$+(\lambda_{nA})P_{2,0,0,0}+\mu_{td}P_{3,0,1,0}=0 \quad \ldots (47)$$

$$-[(\lambda_{nC}+\lambda_h+l\lambda_{nA}+l\lambda_{nB})+3\mu_{td}+\mu_{td}+\mu_{td}]P_{3,0,1,0}+(\lambda_{nC}+\lambda_h+l\lambda_{nA}+l\lambda_{nB})P_{3,0,0,0}$$
$$+2\mu_{td}P_{3,0,2,0}+\lambda_{nA}P_{2,0,1}+\mu_{td}P_{3,1,1,0}=0 \quad \ldots (48)$$

$$-[(\lambda_{nC}+\lambda_h+l\lambda_{nA}+l\lambda_{nB})+3\mu_{td}+2\mu_{td}+(\lambda_{nB}+\lambda_h+l\lambda_{nA}+l\lambda_{nC})]P_{3,0,2,0}+3\mu_{td}P_{3,0,3,0}$$
$$+(\lambda_{nC}+\lambda_h+l\lambda_{nA}+l\lambda_{nB})P_{3,0,1,0}+\mu_{td}P_{3,1,2,0}+\lambda_{nA}P_{2,0,2,0}=0 \quad \ldots (49)$$

$$-[(\lambda_{nB}+\lambda_h+l\lambda_{nA}+l\lambda_{nC})+3\mu_{td}+3\mu_{td}]P_{3,0,3,0}+\mu_{td}P_{3,1,3,0}+\lambda_{nA}P_{2,0,3}$$
$$+(\lambda_{nC}+\lambda_h+l\lambda_{nA}+l\lambda_{nB})P_{3,0,2,0}=0 \quad \ldots (50)$$

$$-[(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})+3\mu_{td}+\mu_{td}]P_{3,1,0,0}+2\mu_{td}P_{3,2,0,0}+\lambda_{nA}P_{2,1,0}$$
$$+(\lambda_{nb}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})P_{3,1,0,0}=0 \quad \text{... (51)}$$

$$-[(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})+3\mu_{td}+2\mu_{td}]P_{3,2,0,0}+3\mu_{td}P_{3,3,0,0}+(\lambda_{nA}+l\,\lambda_h)P_{2,2,0}$$
$$+(\lambda_{nb}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})P_{3,1,0,0}=0 \quad \text{... (52)}$$

$$-[(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})+3\mu_{td}+3\mu_{td}]P_{3,3,0,0}+\mu_{td}P_{3,3,1,0}+(\lambda_{nA}+\lambda_h)P_{2,3,0}$$
$$+(\lambda_{nb}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})P_{3,2,0,0}=0 \quad \text{... (53)}$$

$$-[(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})+3\mu_{td}+\mu_{td}+\mu_{td}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})]P_{3,1,1,0}+2\mu_{td}P_{3,1,2,0}$$
$$+(\lambda_{nA})P_{2,1,1}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})P_{3,0,1,0}+2\mu_{td}P_{3,2,1,0}=0 \quad \text{... (54)}$$

$$-[(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})+3\mu_{td}+\mu_{td}+2\mu_{td}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})]P_{3,1,2,0}+2\mu_{td}P_{3,2,2,0}$$
$$+(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})P_{2,1,2}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})P_{3,0,2,0}+(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})P_{3,1,2,0}$$
$$+3\mu_{td}P_{3,1,3,0}=0$$

... (55)

$$-[\mu_{td}+3\mu_{td}+3\mu_{td}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})]P_{3,1,3,0}+2\mu_{td}P_{3,2,3,0}$$
$$+(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})P_{2,1,3}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})P_{3,0,3,0}+2\mu_{td}P_{3,2,3,0} \quad \text{... (56)}$$
$$+(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})P_{3,1,2,0}=0$$

$$-[\mu_{td}+3\mu_{td}+2\mu_{td}+(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})+3\mu_{td}]P_{3,2,1,0}+2\mu_{td}P_{3,2,2,0}$$
$$+(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})P_{2,2,1}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})P_{3,1,1,0}+(\lambda_{nB}+\lambda_h)P_{3,2,1,0} \quad \text{... (57)}$$
$$+(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})P_{3,2,0,0}=0$$

$$-[2\mu_{td}+3\mu_{td}+2\mu_{td}+(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})]P_{3,2,2,0}+3\mu_{td}P_{3,3,2,0}$$
$$+(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})P_{2,2,2}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})P_{3,1,2,0}+3\mu_{td}P_{3,2,3,0} \quad \text{... (58)}$$
$$+(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})P_{3,2,1,0}=0$$

$$-[3\mu_{td}+3\mu_{td}+2\mu_{td}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})]P_{3,2,3,0}+3\mu_{td}P_{3,3,3,0}$$
$$+(\lambda_{nA}+\lambda_h+l\,\lambda_{nB}+l\,\lambda_{nC})P_{2,2,3}+(\lambda_{nB}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nC})P_{3,1,3,0}+(\lambda_{nC}+\lambda_h+l\,\lambda_{nA}+l\,\lambda_{nB})P_{3,2,2,0}=0$$

... (59)

$$-[3\mu_{td} + 3\mu_{td} + \mu_{td} + (\lambda_{nC} + \lambda_h + l\,\lambda_{nA} + l\,\lambda_{nB})]P_{3,3,1,0} + 2\mu_{td}P_{3,3,2,0} + (\lambda_{nA} + \lambda_h)P_{2,3,1,0} + (\lambda_{nB} + \lambda_h + l\,\lambda_{nA} + l\,\lambda_{nC})P_{3,2,1,0} + (\lambda_{nC} + \lambda_h + l\,\lambda_{nA} + l\,\lambda_{nB})P_{3,3,0,0} = 0 \quad \ldots(60)$$

$$-[3\mu_{td} + 3\mu_{td} + 2\mu_{td} + (\lambda_{nC} + \lambda_h + l\,\lambda_{nA} + l\,\lambda_{nB})]P_{3,3,2,0} + 3\mu_{td}P_{3,3,3,0} + (\lambda_{nA} + \lambda_h + l\,\lambda_{nB} + l\,\lambda_{nC})P_{2,3,2,0} + (\lambda_{nB} + \lambda_h + l\,\lambda_{nA} + l\,\lambda_{nC})P_{3,2,2,0} + (\lambda_{nC} + \lambda_h + l\,\lambda_{nA} + l\,\lambda_{nB})P_{3,3,1,0} = 0 \quad \ldots(61)$$

$$-[3\mu_{td} + 3\mu_{td} + 3\mu_{td}]P_{3,3,3,0} + (\lambda_{nA} + \lambda_h + l\,\lambda_{nB} + l\,\lambda_{nC})P_{2,3,3,0} + (\lambda_{nB} + \lambda_h + l\,\lambda_{nA} + l\,\lambda_{nC})P_{3,2,3,0} + (\lambda_{nC} + \lambda_h + l\,\lambda_{nA} + l\,\lambda_{nB})P_{3,3,2,0} = 0 \quad \ldots(62)$$

$$-[(3M\,\mu_{td} + (r+1)\mu_{tw})]P_{3,3,3,r} + (\lambda_h)P_{3,3,3,r-1} = 0 \qquad 1 \le r \le Q \quad \ldots(63)$$

5. Performance Measures

Many interesting performance indices can be computed using the steady state probabilities. The most important performance measure for our systems is the probability that a call is blocked due to lack of available resources. Using probabilities obtained from equations given in previous section, we can establish various performance measures as follows

- The blocking probability of the new calls in cell A is obtained by

$$B_{nA} = \sum_{s=0}^{Q} P_{M,M,M,s} + \sum_{j=0}^{M-1} P_{M,j,M,0}(1-l) + \sum_{k=0}^{M-1} P_{M,M,k,0}(1-l) \quad \ldots(64)$$

- The blocking probability of the new calls in cell B is obtained by

$$B_{nB} = \sum_{s=0}^{Q} P_{M,M,M,s} + \sum_{i=0}^{M-1} P_{i,M,M,0}(1-l) + \sum_{k=0}^{M-1} P_{M,M,k,0}(1-l) \quad \ldots(65)$$

- The blocking probability of the new calls in cell C is obtained by

$$B_{nC} = \sum_{s=0}^{Q} P_{M,M,M,s} + \sum_{i=0}^{M-1} P_{i,M,M,0}(1-l) + \sum_{j=0}^{M-1} P_{M,j,M,0}(1-l) \quad \ldots(66)$$

- The blocking probability of the new calls in handoff area and handoff calls is obtained by

$$B_h = P_{M,M,M,Q} \quad \ldots(67)$$

- The total carried load T_{ch} in cell A, cell B and cell C is obtained by

$$T_{ch} = \sum_{i=0}^{M} \sum_{j=0}^{M} \sum_{k=0}^{M} (i + j + k) P_{i,j,k,0} + \sum_{s=1}^{Q} 2MP_{M,M,M,s} \qquad (68)$$

6. Successive Overrelaxation Method for a Linear System

Successive overrelaxation method is an iterative technique that uses successive approximations to obtain more accurate solutions to a linear system at each step. Consider a system of n linear equations Ax=b or

$$\sum_{j=1}^{n} a_{i,j} x_j = b_i \text{ , i= 1,2,.........,n}$$

The pseudo code for the successive overrelaxation algorithm is given as follows:

Algorithm:

Input: omega, nmax, toll, err, iter.

Define: r=b-a*x_0

r_0=norm(r)

err= norm(r)

xold=x_0

Choose an initial guess x_0 to the solution x.

iter=0 while err>toll & iter<nmax

iter=iter+1 for i=1,2,-------,n

s=0 for j=1,2,-------,i-1

s=s+$a_{i,j}x_j$ end for j=i+1,i+2,-----,n

s=s+$a_{i,j}$*xold_j end x_i=omega*(b_i-s)/$a_{i,j}$+(1-omega)*xold_i

end x=x(:) xold=x r=(b-a)*x

err=norm(r)/r_0 end

end The "fsolve" function of MATLAB implements the successive overrelaxation method.

7. Numerical illustrations

In this section, we obtain the numerical results for the blocking probabilities of the new calls in cell A, cell B and cell C and the blocking probability of the calls in handoff area by using the MATLAB software. The performance measures are obtained by varying various parameters namely arrival rate $□_{nA}$, $□_{nB}$, $□_{nC}$ and service rate □□in□□Figs 1(a-b).□□ The default parameters are set as N=10, Q=5, /=0.3, □=0.5, $□_{nA}$=0.2, $□_{nB}$ =0.3,□$□_{nC}$ =0.4.

Figs 1(a,b) and 2(a,b) show the effect of the total number of channels N in cells A ai.. cell B. In figs 1(a) and 2(a), it can be noticed that the blocking probabilities of new calls in cell A and B show increasing trends with the arrival rate $□_{nA}$ and $□_{nB}$, respectively. Figs 1(b), 2(b) demonstrate the trends of blocking probability of handoff calls B_h by varying arrival rates $□_{nA}$ and $□_{nB}$, respectively. Further, it is noticed that as we increase N, the blocking probability of handoff calls B_h decreases.

Figs 3(a,b) show the blocking probabilities of new and handoff calls in cell C on varying the new arrival rate $□_{nC}$ for different values of total number of channels N=10-14. We note that B_{nC} increases when $□_{nC}$ increases. It is easily observed from the graph that B_h initially decreases gradually and afterwards rapidly on increasing $□_{nC}$.

Figs 4(a-c) exhibit the blocking probabilities of new calls in cell A, B and C for different values of service rate □. The blocking probabilities of new calls B_{nA}, B_{nB} and B_{nC} decrease rapidly with the increases in □□

In conclusion, we can say that it offers the least blocking of the handoff calls as well as to the new calls can be achieved by the choice of suitable service rate. Numerical resu.. provided demonstrate the computational tractability of the analytical results as well as give insight how the grade of service (GoS) of the system can be insured.

8. Conclusion

We presented the modeling and performance evaluation in the integrated ad hoc and cellular network system. The newness in the model is that we able to improve the performance in terms of performing experimental analysis on the preferred traffic model of priority scheme. The integrated system with the handoff area which is unstable, complicated and very important w.. modeled and analyzed. The novel approach is based on the development of model whose state spaces grow linearly with the number of calls that can be simultaneously in progress within a cell. We have obtained the probability vectors, which are further used to determine vario... performance measures.

The suggested models are easy to employ and improves the performance of handoff calls but at the cost of new calls as the blocking of new calls increases. The selection of particular handoff calls depends on its implementation complexity and performance. The sensitivity analysis done may be helpful to system engineers during design and development phases. Though, limited number of allocated channels has been considered for simplicity still the model shows satisfactory results in some extend.

The result shows that the Successive Over-Relaxation method is more efficient than the other method. It can be considering their performance, using parameters as time to converge, storage and level of accuracy. Based on our investigation, the suggested model may be implemented to improve the grade of the service (GOS) of the real time system. Further study on the users mobility and presence of more callers in each cell with supplementary channels are in progress.

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Synthesis, Spectral Study, Characterization and Antimicrobial Activity of Zinc (II) Complex of Chalcone of Pyridine-2-Carbaldehyde

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Abstract: ***Metal complex of Zn(II) has been synthesized with newly prepared biologically active ligand. This ligand was prepared by the Claisen-Schmidt condensation method of 2,6-dihydroxy acetophenone and pyridine-2-Carbaldehyde. The structure of the complex has been proposed by the analytical data, conductivity measurement, magnetic moment, IR spectrum, Electronic absorption spectrum, thermal studies and XRD analysis. Analytical data confirmed 1:2 stoichiometry and the electronic spectral data, IR, magnetic moment, TG-DTA suggests that Zn(II) complex has square planar geometry. Absence of coordinated water molecules in Zn(II) complex is confirmed by TG-DTA studies. The conductivity data show that the complex is non electrolyte. Antimicrobial activities of complex with selected bacterial strain and fungal strain carried out and the results have been compared with commercial standards. In this paper we prepare chalcone of pyridine-2-Carbaldehyde by Claisen-Schmidt condensation method and synthesize Zn(II) metal complex and characterize them by Infrared, Electronic absorption spectra, magnetic susceptibility, CHO analysis, solution conductivity, XRD study, TG-DTA and antimicrobial activity.***

Keywords: ***Electronic absorption spectrum, Infrared spectrum, TG-DTA, Elemental analysis, Antimicrobial activities, Physico-chemical property, Magnetic susceptibility and Conductivity.***

I. INTRODUCTION

Chalcones constitute an important group of natural products, chemically they consist of open chain flavonoids in which the two aromatic rings are joined by α, β unsaturated carbonyl system. The name chalcone is given by Kostanecki and Tambar [1]. Many complexes of chalcones are synthesized and studied in the literature [2-3]. It is believed that the (>CO–C=C<), moiety imparts biological characteristics to this class of compounds. Such α, β-unsaturated carbonyl compounds and their metal complexes possess interesting biochemical properties, such as antitumour, antioxidant, anti-fungal and antimicrobial activities [4]. The electronic absorption spectrum, infrared spectrum, magnetic moment, TG-DTA supports the square planar geometry of the metal complex of chalcone. All crystals of a substance possess the same elements of symmetry. The computer program, used for indexing data was powder-X. Furthermore, biological activities of complex with selected bacterial strain and fungal strain carried out and the results have been compared with commercial standard [5].The X-ray powder diffractogram of the metal complexes was used for the structural characterization and determination of lattice dimensions.

II. MATERIALS AND METHODS

A. *Synthesis of chalcone of Pyridine-2-Carbaldehyde*

The reagents used for preparation of chalcone of pyridine-2-carbaldehyde are of A.R. grade. 2,6-dihydroxy acetophenone (0.01mol) and pyridine-2-carbaldehyde (0.01mol) are dissolved in ethyl alcohol (25 mL) and then potassium hydroxide 10 mL (40%) were added to it. The reaction mixture was heated for 3 hours till yellow-brown color was obtained. The progress of reaction was monitored by TLC, after completion of reaction the content was poured into ice cold water and then acidified by dil. HCl the solid obtained was filtered and the crude product was recrystallized from ethyl alcohol to give pyridine chalcone[6].

B. *Synthesis of metal complex*

The solution of 0.02 mole of chalcone of pyridine-2-carbaldehyde was taken in round bottom flask containing 30 ml of anhydrous methanolic solution and boiled for 10 minutes. A hot solution of 0.01 mole, of Zinc Nitrate in 20 ml of methanol was added drop wise to the solution of the chalcone of pyridine-2-carbaldehyde To this reaction mixture, 10% alcoholic ammonia was added up to slightly alkaline pH. The complex was precipitated at 9 pH range. The pH 8-10 range was definite for these complexes [7]. The

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contents were stirred on magnetic stirrer for one hour. The solid metal complex separated out and washed with methanol three to four times. Dried in vacuum desiccators over anhydrous granular calcium chloride. The melting point/decomposition a temperature of the complex was determined by Thiele's melting apparatus. The reactions of formation of Zn(II) complex is shown in figure (1).

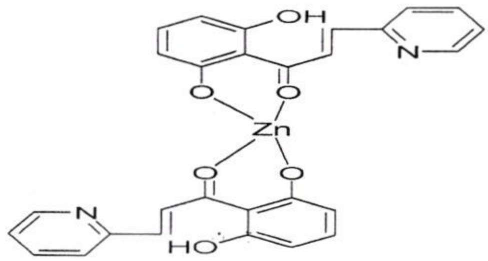

Fig.(1): Metal complexes of Zinc (II) with chalcone of pyridine-2-carbaldehyde

III. RESULTS AND DISCUSSION

A. *Infra Red Spectrum*

1) *Infrared spectrum of Zn(II) complex:* The infrared spectra of ligands and Zn(II) complex of chalcone of pyridine-2-carbaldehyde was recorded on a Perkin-Elmer Spectrum RX-IFTIR Spectrophotometer over the range 4000-400 cm^{-1} using KBr pellet at CIL, Chandigarh, Punjab.

Seema Habib assigned [8] the ligand which shows a weak broad band around 3047-3029 cm^{-1}. In Zn complexes do not show any absorbance for –OH of the coordinated water molecule. In the IR spectra of all ligands, an intense band appearing around 1656 cm^{-1} is attributed to C=O group. The medium intensity band appearing around 1530 cm^{-1} in the ligand and the complexes are assigned to C=O aromatic. The M-O band for Zn(II) was observed at 500 cm^{-1}.

Chiara Sulpizio assigned that, [9] Zn^{+2} do not show any significant shift compared to the free ligand. While the Zn^{+2} complexes show a shift of C=0 stretching mode from 1630 to 1610 cm^{-1} indicating coordination of carbonyl oxygen to a metal ion.

In the chalcone of pyridine-2-carbaldehyde a sharp strong band observed at 1597 cm^{-1} is attributed to (C=C) stretching mode in the IR spectra of ligand. This band observed at 1437 cm^{-1} in Zn(II) complexes. The slight shift in the band of (C=C) stretching frequency is due to change in electron distribution across (C=C) bond in the metal complexes[10].

The presence of phenolic –OH is confirmed by peaks at 3055 cm^{-1} in the ligand, In the spectra of Zn(II)) complexes, there is the complete disappearance of the peak at 3055 cm^{-1} in chalcone which suggests absence of phenolic group –OH indicates its coordination. (C-O-C) is shifted to a lower wave number compared with a free ligand. In the IR spectra of ligand, the strong bands appeared in the region 1623 & 1666 cm^{-1} are assigned to υ C=O) of stretching frequency [11]. It is shifted towards lower frequency than corresponding ligands and appeared at 1599 cm^{-1} in metal complexes. Such a lowering in stretching vibration of υ (C=O) in chalcone indicates the participation of chalcone carbonyl in complexation. In the IR spectra of Zn(II) complexes, In Zn(II) complexes new band is observed at 507 cm^{-1} due to the (M-O) bond.

Table no. (1): Infrared absorption frequencies (cm^{-1}) of chalcone of pyridine-2-carbaldehyde Zn(II) complex.

Ligand/ Metal complexes	υ(OH) cm^{-1}	υ (H_2O) cm^{-1}	υ ((-CO-CH=CH-))cm^{-1}	υ ((C-O-C) cm^{-1}	υ (C=C) cm^{-1}	Aromatic ring (C=C)cm^{-1}	υ(M-O)cm^{-1}
chalcone of pyridine-2-carbaldehyde	3055		1666	1099	1597	1435	--
[Zn (chalcone of pyridine-2carbaldehyde)2]	3068	-	1599	1052	1437	1369	507

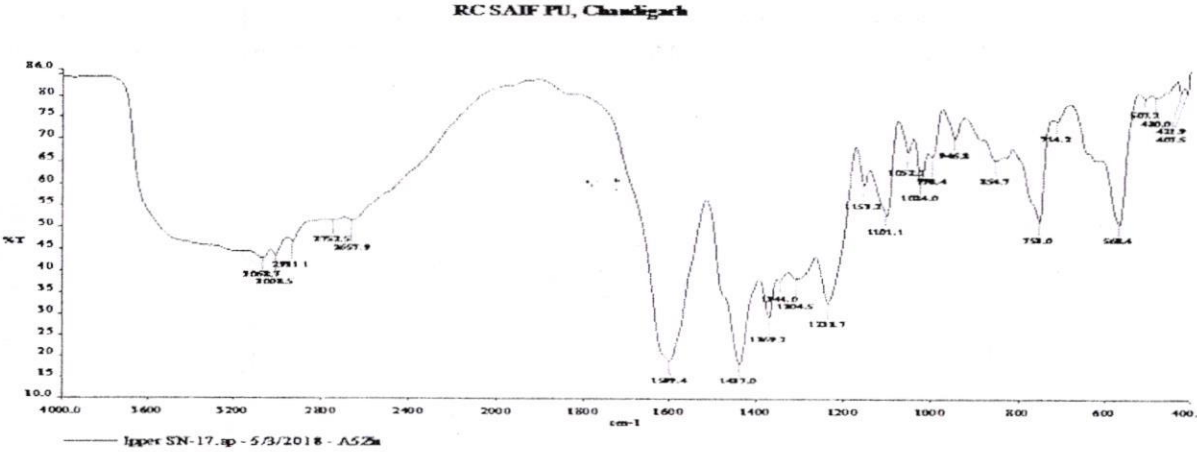

Fig. (2): Infrared spectra of Zn metal complex.

B. *Physical Parameters*

Table no.(2): Physical parameters of Zinc (II) complex.

Metal complex	Ligand	p^H range ppt	Color	M.P. ^{0}C
Zinc(II) complex	chalcone of pyridine-2-carbaldehyde	7.0-7.5	Reddish brown	280

C. *CHO Analysis*

The carbon, hydrogen, oxygen, cobalt and copper metals percentage in Zn(II) complex of chalcone measured at SAIF Cochin, Kerala. The calculated and measured values of CHO analysis are matching and are given in the table no.(3).

Table no. (3): CHO analysis

Metal complexes	Chemical formula	Mol. Wt.	Elemental analysis : % found (calculated)						
			C	H	N	O	S	X(Br)	M
Zn (II) complex	$[C_{28}H_{20}O_6N_2Zn]$	545	61.61 (57.80)	3.69 (4.16)	5.13 (4.81)	17.58 (21.99)	-	-	11.90 (11.23)

D. *Magnetic Susceptibility, Solution Conductivity And Electronic Absorption Spectral Data*

1) *Magnetic Susceptibility:* The μ_{eff} (B.M.) values at room temperature for Zn(II) complex is dimagnetic these values agree with Square planar geometry of the metal complex [12-13].
2) *Solution Conductivity:* The solution conductivities of 10^{-3} M solution of metal complex in DMSO were measured on EQUIPTRONICS digital conductivity meter EQ - 660 with 20 μΩ to 200 μΩ at 298K temperature. In the present investigation Zn(II) complex is reddish brown in color, stable to air and moisture. Decomposes at high temperature rather than showing sharp melting points. They are insoluble in water and soluble in DMSO, DMF. The low conductivity values in DMSO solution (10^{-3} M) are given in table no.(4) indicates non-electrolytic nature.
3) *Electronic Absorption Spectral Study:* Electronic absorption spectrum was measured on SL159, single beam UV-VIS spectrophotometer.

The electronic spectrum of Zn(II) complex studied in the present investigation exhibit absorption band at 27247(367 nm) cm^{-1} which are assigned to charge transfer band.

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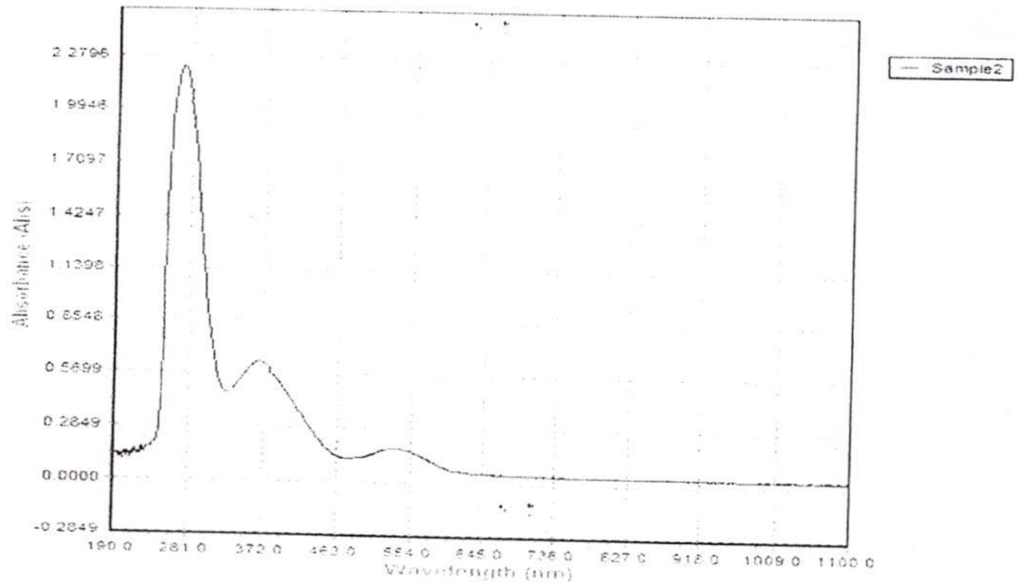

Fig. (3): Electronic absorption spectra of Zn (II) complex of chalcone of pyridine-2-carbaldehyde

Table no. (4): Solution conductivity, magnetic and electronic absorption spectral data of Zn(II) complex.

Zn(II) Complex	Molar Conductance $Ohm^{-1}cm^2mol^{-1}$	μ_{eff} (B.M.)	Absorption Maxima (LMCT) cm^{-1} Charge transfer.
Chalcone of pyridine-2-carbaldehyde	8.04	Diamagnetic	27247(367)

E. Thermal Analysis Zn(II) Complex Chalcone of Pyridine-2-Carbaldehyde

The simultaneous thermogravimetric, differential thermal analysis of Zn(II) complex chalcone of pyridine-2-carbaldehyde was performed in an inert nitrogen atmosphere on Perkin Elmer STA 6000 at SAIF, Cochin, Kerala. The heating rate was 10°/min and flow rate of nitrogen 50 ml/min. The reference substance used was α Al_2O_3 in platinum crucible and sample weighed in the range of 4-12 mg. The thermogram of Zn(II) complex of chalcone of pyridine-2-carbaldehyde is presented in figure (4). This curve reveals that there is absence of lattice as well as coordinated water in the complex.

The TG-DTA curve of Zn(II) complex chalcone of pyridine-2-carbaldehyde reveals that the complex is thermally stable decomposes above 250°C and there is no weight loss up to 250°C indicating the absence of lattice water as well as coordinated water. The first step shows rapid decomposition within a temperature range 250-330°C with a weight loss of 22.67% (calc. wt. loss 23.79%) which may be due to loss of non-coordinated part two pyridine ring fragments of the ligand. This is confirmed by an endothermic peak at 293.00°C in DTA. The second step decomposition with a weight loss of 36.05% in the range 340-480°C, corresponds to the decomposition of coordinated part of the complex. This is confirmed by an endothermic peak at 414.05°C in DTA. The third step decomposition with a weight loss of 28.98% in the range 490-600°C, corresponds to the decomposition of remaining coordinated part of the complex. This is confirmed by an endothermic peak at 557.36°C in DTA. The compound finally decomposes above 600 °C with a weight loss 14.92% and form ZnO as final product.

The thermal behavior of zinc metal complexes in the present study indicates high thermal stability. Decomposition of all the complexes is started at a relatively higher temperature (~250°C), finally giving a metal oxide residue. Thermograms of all the Zn complexes indicate the absence of lattice water as well as coordinated water molecules and it exhibit higher thermal stability.

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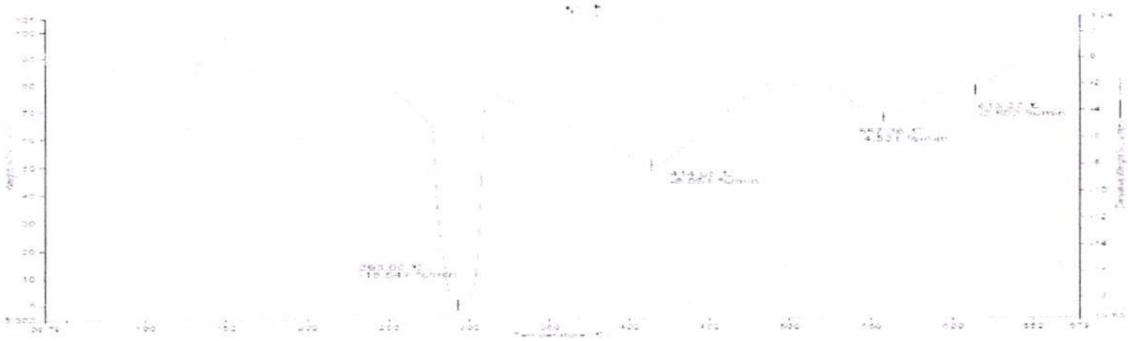

Fig. (4): TG-DTA curve of Zn(II) complex of chalcone of pyridine-2-carbaldehyde

F. X-Ray Diffraction Spectral Studies Of Metal Complex Of Zn(Ii) Complex Of Chalcone Of Pyridine-2-Carbaldehyde

The XRD spectral study has been done at SAIF, Cochin, Kerala. The X-ray diffraction patterns of Zn (II) complex is shown in (Fig .5). The standard deviation observed is within the permissible range. [14] The observed and calculated densities of Zn(II) complex of chalcone of pyridine-2-carbaldehyde are 1.909 gcm^{-3} and 1.816 gcm^{-3} respectively. Chalcone of pyridine-2-carbaldehyde was of Zn(II) complex is found to be tetragonal lattice type with space group P2/m and lattice parameters are a (Å) = 4.9162 b (Å) = 4.9162 c (Å) = 5.4089 $\alpha = 90^\circ$ $\beta = 90^\circ$ $\gamma = 90^\circ$ unit cell volume (V) =328.46 $Å^3$

1) Unit cell data and crystal lattice parameters for Co(II)

Unit cell data and crystal lattice parameters

a (Å) = 4.9162 b (Å) = 4.9162 c (Å) = 5.4089 $\alpha = 90^\circ$ $\beta = 90^\circ$ $\gamma = 90^\circ$ Volume (V) =328.46 $Å^3$

Density (obs.) = 1.2196 gcm^{-3} Density (cal.) = 1.1265 gcm^{-3} Z = 4 Crystal system= Tetragonal

Space group = P2/m Standard deviation (%) = 0.027

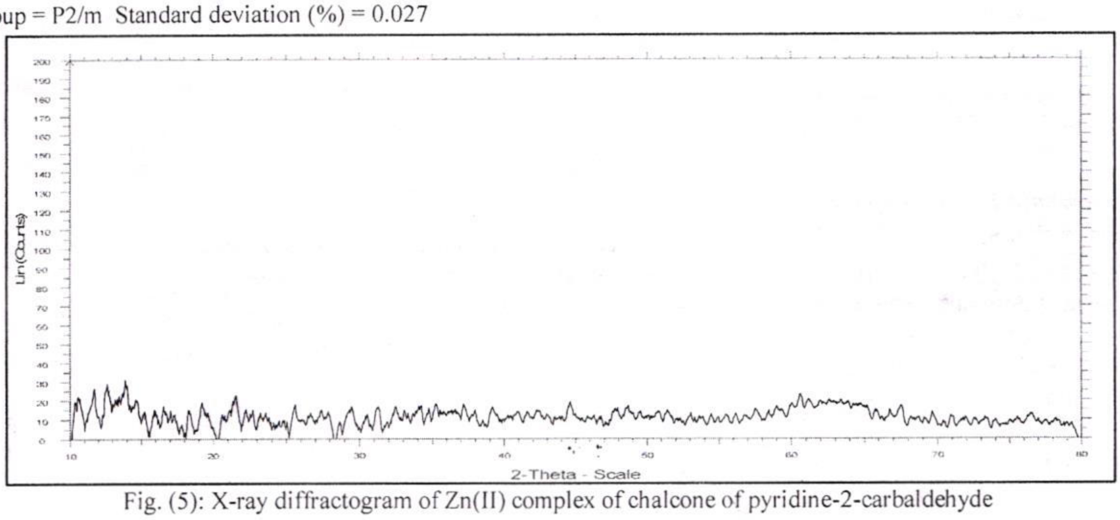

Fig. (5): X-ray diffractogram of Zn(II) complex of chalcone of pyridine-2-carbaldehyde

G. Antimicrobial Activity

Antimicrobial activity was assayed by cup plate agar diffusion method [15] by measuring inhibition zones in mm. In vitro antimicrobial activity of all synthesized compounds and standard have been evaluated against strains of The fungal toxicity of Zn(II) complex was studied in vitro against Aspergillus niger ATCC 16404, Saccharomyces cerevisiae ATCC 9763, Candida albicans ATCC10231 fungal pathogens at fixed 1% concentration. The antibacterial activity of Zn(II) complex was studied, for evaluating antibacterial activity Gram positive and Gram negative bacterial pathogens were used. Staphylococcus aureus ATCC 6538, Bacillus megaterium ATCC 2326, Bacillus subtilis ATCC 6633 were Gram positive pathogens used in this study. Escherichia coli ATCC8739, Salmonella typhi ATCC9207, Shigella boydii ATCC 12034, Enterobacter aerogenes ATCC13048, Pseudomonas aerogenosa ATCC9027, Salmonella abony NCTC6017 were the Gram-negative pathogens used in this stud. From the results of antimicrobial activity of ligands and complexe it is clear that the Zn(II) complex shows enhanced activity than ligand. The increase in antimicrobial activity is due to faster diffusion of metal complex as a whole through the cell membrane or due to the combined activity of the metal and ligand.

IV. CONCLUSION

The Zn (II) complex was colored, insoluble in most of the organic solvent but soluble in organic solvent. The stoichiometry ratio of the metal complexes obtained has been found to be 1:2.Solution conductivity of this metal complex reveals nonelectrolytic nature. The infrared spectral data indicate that the ligand act as mononegative bidentate species towards Zn(II) complex. The electronic spectral data, IR spectrum, magnetic moment, TG-DTA suggests that Zn(II) has Square planar geometry. The CHO analysis gives C, H, and O percentage in the metal complex. The XRD parameters shows that the structure of Zn (II) is tetragonal and has space group = P2/m.

V. ACKNOWLEDGEMENT

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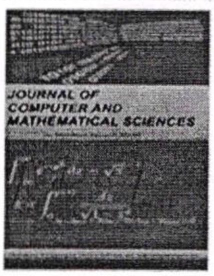

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Zagreb Polynomials of Nanostar Dendrimers

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ABSTRACT

Let G be a graph with vertex set V(G) and edge set E(G). Graph polynomials are polynomials assigned to molecular graphs. Dendrimers are a class of molecules with a well-defined tree-like architecture. Dendrimers are a novel class of spheroid/globular nanoscaled macromolecules. In this paper degree-based and eccentricity-based Zagreb polynomials of nanostar dendrimers are investigated.

Keywords: Degree, dendrimers, eccentricity, molecular graph, Zagreb polynomial.

INTRODUCTION

Let $G = (V, E)$ be a connected graph with the vertex set $V = V(G)$ and the edge set $E = E(G)$, without loops and multiple edges. The number of vertices of G adjacent to a given vertex v, is the degree of this vertex and will be denoted by $d_v(G)$ or d_v. Most of the research in the study of molecular descriptors is on distance and degree-based topological indices. A Zagreb index of nanostructures is vital area in the study of molecular topology. Zagreb indices may be degree-based or eccentricity-based. Zagreb indices can be computed from the corresponding Zagreb polynomials for the molecular graph. The nanostar dendrimer is a part of a new group of macromolecules that seem photon funnels just like artificial antennas and also a great resistant of photo bleaching[1]. The nanostar dendrimers are being investigated for possible uses in nanotechnology, gene therapy, and other fields[2]. M-polynomials can be studied for the degree and eccentricity of molecular graphs. Dendrimers are repeatedly branched roughly spherical large molecules and possess well defined chemical structures[3]. A topological index for a molecular graph G is a numerical value for correlation of chemical structure with various physical properties, chemical reactivity or biological activity[4]. H.Hosoya introduced the Wiener polynomial (Hosoya polynomial) in 1988[5]. The Hosoya polynomial is defined as:

$H(G,x) = \frac{1}{2} \sum_{uv \in E(G)} x^{d(v,u)}$

Where d(v,u) is distance between the vertices u and v. The Zagreb polynomials are studied for different molecular graphs in[6-11].The degree-based Zagreb polynomials $M_1(G,x)$ and $M_2(G,x)$ are defined as:

$M_1(G,x) = \sum_{uv \in E(G)} x^{d_u+d_v}$

$M_2(G,x) = \sum_{uv \in E(G)} x^{d_u d_v}$

and $M_3(G,x)$, $M_4(G,x)$ and $M_5(G,x)$ are[12]:

$M_3(G,x) = \sum_{uv \in E(G)} x^{|d_u-d_v|}$

$M_4(G,x) = \sum_{uv \in E(G)} x^{d_u(d_u+d_v)}$

and $M_5(G,x) = \sum_{uv \in E(G)} x^{d_v(d_u+d_v)}$

where d_v is the degree of vertex v. The eccentricity based topological indices are studied by[13]. The eccentric–connectivity polynomial for $D_1[n]$ with n = 2 is studied by Soleimani[14]. In this paper some degree-based and eccentricity-based Zagreb polynomials are investigated for nanostar dendrimers.

MATERIALS AND METHODS

A molecular graph is constructed by representing each atom of a molecule by a vertex and bonds between atoms by edges. The degree of each vertex equals the valence of the corresponding atom. If the total number of vertices V(G) and total number of edges in 2-dimensional graph are known for a nonmaterial then the topological polynomials and the corresponding topological indices can be computed. Graph polynomials are the polynomials assigned to molecular graphs. The fourth Zagreb polynomial can be degree-based or eccentricity-based. The sixth Zagreb polynomial is eccentricity-based polynomial. Eccentricity is the largest distance between v and any other vertex u of G. In the study of degree-based topological polynomials the vertex-degree of each vertex has to be decided first and in eccentricity-based topological polynomials the eccentricity, the eccentricity of a vertex $u \in V(G)$ is denoted by ec(u) and is defined as:

$ec(u)=\max\{d(u,v)| v \in V(G)\}$.

The fourth and sixth Zagreb polynomials can be defined as:

$Z_{g4}(G,x) = \sum_{uv \in E(G)} x^{(ec(u)+ec(v))}$

and $Z_{g6}(G,x) = \sum_{uv \in E(G)} x^{((ec(u)\, ec(v))}$

where ec(u) is the eccentricity of vertex u in a molecular graph.

RESULTS AND DISCUSSION

The degree is defined as the number of edges with that vertex. A vertex with degree 1 is called pendent vertex. The degree-based Zagreb polynomials are studied for nanostar dendrimer $NS_1(n)$-polypropylenimine in[15]. In[16] the $M_1(G,x)$, $M_2(G,x)$ and $M_3(G,x)$ are defined and studied for Nicotine. The degree-based Zagreb polynomials $M_1(G,x)$ - $M_5(G,x)$ are discussed in this section for nanostar dendrimers.

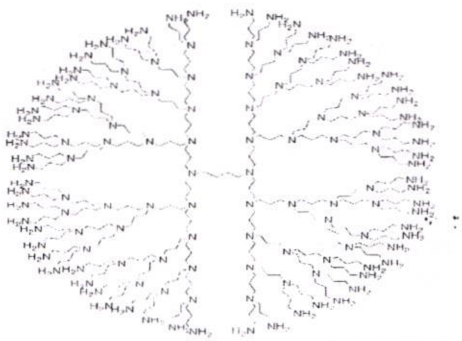
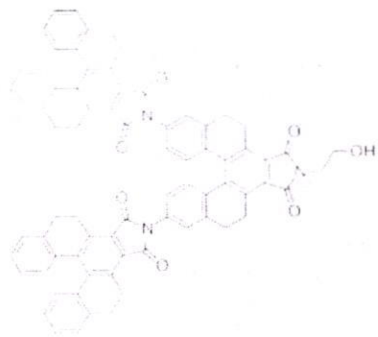

Fig.1.The structure of $NS_2[n]$ and Fig.2. The Wang's Helicene-based dendrimer D_n for n = 2.

The edge partitions for nanostar dendrimer $NS_2(n)$ polypropylenimine are given in table (1).The Zagreb polynomials are,

$M_1(G,x) = \sum_{uv \in E(G)} x^{d_u+d_v}$

$M_2(G,x) = \sum_{uv \in E(G)} x^{d_u d_v}$

In [12] the following Zagreb polynomials are defined: $M_3(G,x)$, $M_4(G,x)$ and $M_5(G,x)$ as:

$M_3(G,x) = \sum_{uv \in E(G)} x^{|d_u-d_v|}$

$M_4(G,x) = \sum_{uv \in E(G)} x^{d_u(d_u+d_v)}$

And $M_5(G,x) = \sum_{uv \in E(G)} x^{d_v(d_u+d_v)}$

where d_u is degree of vertex u in G. The Zagreb polynomials $M_1(G,x)$-$M_5(G,x)$ are computed as: From the structure of nanostar dendrimer $NS_2(n)$ polypropylenimine the edge partitions E(G) are $E_{(1,2)}$,$E_{(2,2)}$ and $E_{(2,3)}$.

$E_{(1,2)} = \{e = uv \in E(G) | du = 1 \& dv = 2\} \rightarrow |E_{(1,2)}| = 2*2^n$,

$E_{(2,2)} = \{e = uv \in E(G) | du = 2 \& dv = 2\} \rightarrow |E_{(2,2)}| = 8*2^n - 5$,

$E_{(2,3)} = \{e = uv \in E(G) | du = 2 \& dv = 3\} \rightarrow |E_{(2,3)}| = 6*2^n - 6$.

Table no.1. The edge partitions E(G) for nanostar dendrimer $NS_2(n)$ polypropylenimine.

$d_u,d_v/uv \in E(G)$	(1,2)	(2,2)	(2,3)
Number of edges	$2*2^n$	$8*2^n$-5	$6*2^n$-6

It is observed from figure (1) that |V| = 90, and |E| = 102 for nanostar dendrimer $NS_2(n)$ polypropylenimine. The first and second Zagreb polynomials can be computed as:

$M_1(G,x) = \sum_{uv \in E(G)} x^{d_u+d_v}$

$= \sum_{E_1=12 \in E(G)} x^{d_u+d_v} + \sum_{E_2=22 \in E(G)} x^{d_u+d_v} + \sum_{E_3=23 \in E(G)} x^{d_u+d_v}$

$= \sum_{E_1=12 \in E(G)} x^{1+2} + \sum_{E_2=22 \in E(G)} x^{2+2} + \sum_{E_3=23 \in E(G)} x^{2+3}$

$=2*2^n x^3 + (8*2^n-5)x^4+(6*2^n-6)x^5$.

and

$M_2(G,x) = \sum_{uv \in E(G)} x^{d_u d_v}$

$= \sum_{E_1=12 \in E(G)} x^{d_u d_v} + \sum_{E_2=22 \in E(G)} x^{d_u d_v} + \sum_{E_3=23 \in E(G)} x^{d_u d_v}$

$= \sum_{E_1=12 \in E(G)} x^{1*2} + \sum_{E_2=22 \in E(G)} x^{2*2} + \sum_{E_3=23 \in E(G)} x^{2*3}$

$=2*2^n x^2 + (8*2^n - 5) x^4 + (6*2^n - 6) x^6$.

The computed degree-based Zagreb polynomials $M_1(G,x)$-$M_5(G,x)$ are given in table (2).

The eccentricity of a vertex v in a graph G, denoted ec(v), is the distance from v to a vertex farthest from v that is

$ec(v) = \max_{x \cdot VG} \{d(v,x)\}$.

The fourth and sixth Zagreb polynomials can be defined[17] as:

$Z_{g4}(G,x) = \sum_{uv \in E(G)} x^{((ec(u) + ec(v))}$ and sixth Zagreb polynomial

$Z_{g6}(G,x) = \sum_{uv \in E(G)} x^{((ec(u)\, ec(v))}$.

By the structure analysis of molecular structure of D(n),the edge set can be divided into n parts as:

E_1:ec(u) = n and ec(v) = n+1;

E_2:ec(u) = n+1 and ec(v) = n+2;

E_i:ec(u) = n+i-1 and ec(v) = n+i;

E_n:ec(u) = 2n-1 and ec(v) = 2n.

The eccentricity based Zagreb polynomials are computed as: It is observed from figure number (2) for Wang's Helicene-based dendrimer D_n with n=2, |V|=82 and |E|=99.

The fourth Zagreb polynomial is:

$Z_{g4}(G,x) = \sum_{uv \in E(G)} x^{(ec(u) + ec(v))}$

$=(2*2^2-1)E_{17,18} + (2^2-2)(E_{17,17}+E_{23,23}+E_{18,19}+E_{21,21}) + (3*2^2-2)(E_{16,17})+ (2^2-3)(E_{16,16}+E_{14,14}) + 2*2^2(E_{15,16}+E_{20,21}+E_{22,23}+E_{23,24}+E_{25,26}+E_{26,27}) +2^2(E_{14,15}+E_{19,20}+E_{21,22})+ 3*2^2(E_{24,25})$.

$=8x^{53}+8x^{51}+12x^{49}+8x^{47}+2x^{46}+8x^{45}+4x^{43}+2x^{42}+8x^{41}+4x^{39}+2x^{37}+7x^{35}+2x^{34}+10x^{33}+x^{32}+8x^{31}+4x^{29}+x^{28}$

The fourth Zagreb polynomial and sixth Zagreb polynomial are computed for nanostar dendrimers and represented in table number (2). It is easy to see that the Zagreb indices of a graph can be obtained from the corresponding Zagreb polynomials by evaluating their first derivatives at x=1. For example the fourth Zagreb index is:

$Z_{g4}(G) = \frac{\partial Zg4(G,x)}{\partial x}/_{x=1} = 4129$.

Table no. 2. Zagreb polynomials for nanostar dendrimers.

Polynomial	Zagreb polynomial
$M_1(G,x)$	$(6*2^n-6)x^5+(8*2^n-5)x^4+2*2^n x^3$.
$M_2(G,x)$	$(6*2^n-6) x^6 + (8*2^n-5)x^4+2*2^n x^2$.
$M_3(G,x)$	$(8*2^n-6) x^1 + (8*2^n-5)$.
$M_4(G,x)$	$(6*2^n-6)x^{10}+(8*2^n-5)x^8+(2*2^n)x^3$.
$M_5(G,x)$	$(6*2^n-6) x^{15}+ (8*2^n-5)x^8+(2*2^n)x^3$.
$Z_{g4}(G,x)$	$8x^{53}+8x^{51}+12x^{49}+8x^{47}+2x^{46}+8x^{45}+4x^{43}+2x^{42}+8x^{41}+4x^{39}+2x^{37}+7x^{35}+2x^{34}+10x^{33}+x^{32}+8x^{31}+4x^{29}+x^{28}$
$Z_{g6}(G,x)$	$8x^{702}+8x^{650}+12x^{600}+8x^{552}+2x^{529}+8x^{506}+4x^{462}+2x^{441}+8x^{420}+4x^{380}+2x^{342}+x^{306}+2x^{289}+10x^{272}+x^{256}+8x^{240}+4x^{210}+x^{196}$.

CONCLUSION

In this paper degree-based and eccentricity-based Zagreb polynomials are studied for nanostar dendrimers. The Zagreb indices can be computed from the corresponding Zagreb polynomials. The Zagreb indices are very useful in QSAR/QSPR study.

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ગુજરાત સંશોધન મંડળનું ત્રૈમાસિક

JOURNAL OF THE
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Synthesis, spectral Characterization and Antimicrobial Activity of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one

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Abstract

Flavonoids are also known as plant pigments or co-pigments. Flavonoids are the largest group of naturally occurring phenolic compounds which occur in different plant parts both in a free state and as glycosides. 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one was synthesized by Claisen-Schmidt condensation method from 2,6-dihydroxy acetophenone and 2-furaldehyde to gives chalcone and which on oxidation with DMSO/I2.Spectroscopic characterization using UV-visible, IR, ^{1}H NMR, Mass spectra, properties and antimicrobial activity has been investigated for 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one. The IR bands for Carbonyl group (-C=O in pyron ring) for 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one appear at 1605 per cm which agrees with the general range of flavones. The 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one has two matching bands at 245 nm and other at 360 nm which lies in the range of flavone. In the mass spectrum of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one molecular ion peak is observed m/z and calculated m/z corresponding to $[M^+]$ peaks is in good agreement with their structure. The flavone shows moderate to good Antibacterial and Antifungal activity.

Keywords: *2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one, UV-visible, IR, ^{1}H NMR, Mass spectrum, CHO analysis and Antimicrobial Activity.*

1. Introduction

The flavonoids or bioflavonoids are a ubiquitous group of polyphenolic substances which are present in most plants. They also occur as glycosides. Chemically, flavonoids show a fifteen-carbon skeleton (C6-C3-C6) which consists of two phenyl rings connected by a three carbon bridge. Flavonoids are found to occur in different parts of the plant roots, barks, wood, leaves, flowers, fruits, and seeds. Flavonoids are synthesized in all parts of the plants. They play role in providing color, fragrance and taste to the fruits, flowers and seeds, which makes them

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attractants for insects, birds or mammals, which aid in pollen or seed transmission.[1] Flavonoids lead to potent antioxidant activity, the most important function of flavonoids to scavenge hydroxyl radicals, superoxide anions and lipid peroxy radicals. Multiple combinations of hydroxyl groups, sugar, oxygen and methyl groups attached to these structures create the various classes of flavonoids, flavonols, flavonones, flavones, flavon-3-ols (catechins), anthocynins and isoflavones.[2] Chalcone is an inter-mediate compound in the biosynthesis of flavonoids, which are the substances widespread in plants and with an array of biological activities. Several therapeutically interesting biological activities of certain flavonoids have been reported including antibacterial,[3] antiviral, anti-inflammatory, antiallergic, antithrombotic,[4] antimutagenic, antineoplastic, neuroprotective properties,[5] and antioxidant properties, etc.

2. Materials and Methods

The chalcone of 2-furaldehyde was synthesized by Claisen-Schmidt condensation of 2,6, di-hydroxyacetophenone (0.01 mol) and 2-furaldehyde (0.01 mol. Which on oxidation with $DMSO/I_2$ it gives flavones. The chemicals used for this synthesis are of AR grade.

Synthesis of Chalcone:

A mixture of 2,6-dihydroxy acetophenone (0.01 mol) and 2-furaldehyde (0.01 mol) are dissolved in ethanol (20 mL) and then solution of potassium hydroxide 10 mL (15%) were added to it. The mixture was stirred for overnight. The progress of the reaction was monitored by TLC. It was then poured on ice cold water and acidified with dilute HCl. The coffee brown solid was precipitates, filtered and washed with water and recrystallized from ethanol.[6]

Synthesis of Flavone:

The chalcone of 2-furaldehyde was dissolved in 20 ml DMSO, to this catalytic quantity of iodine was added. Contents were refluxed for one hour, the progress of the reaction was monitored by TLC and the reaction mixture was left overnight. It was then poured on ice cold water, the separated solid was filtered washed with cold water followed by a dilute sodium-thiosulphate solution. The product was crystallized from ethanol.[7-8]

The melting point of the flavone was determined by an open capillary tube and is unconfirmed. Infra red spectrum was measured using FT-IR spectrophotometer, UV-visible spectrum measured on SL 159 single beam UV-VIS spectrophotometer, ^1H NMR measured on Bruker AVANCE II 400 MHz Spectrophotometer and Mass spectrum was measured on Mass spectrophotometer .The purity the compound was checked by TLC plate which were precoated with silica gel using solvent ethyl acetate and petroleum ether (3:7). The reaction for formation of flavone is given figure (1).

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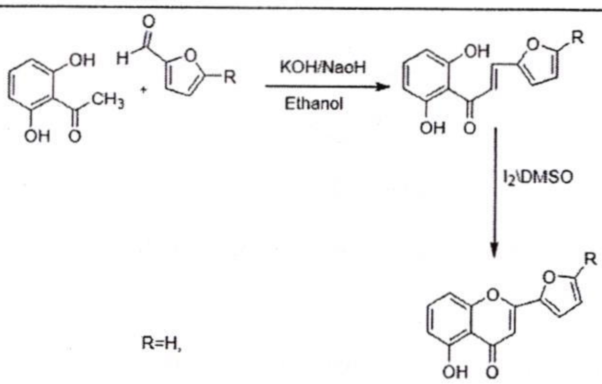

Fig.(1): Reaction of formation of Flavone.

3. Results and discussion

3.1 Properties

The flavone having IUPAC name 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one was synthesized by Claisen-Schmidt condensation method and its structure is stable at room temperature, insoluble in water and is soluble in organic solvent (ethyl alcohol). The physico-chemical properties of flavone and CHO analysis by calculation method are given in table no. (1). The reaction of formation of flavone is shown in figure (1). Elemental analysis showed that the percentage of the Oxygen, hydrogen and carbon found experimentally is equivalent to the calculated values for this compound. The chemical structure of $C_{13}H_8O_4$ is shown in figure (2).

Table no. (1): The properties of flavone and Elemental analysis data.

Mol. Formula	Color	Mol. Wt.	M.P °c	Found (Calculated) %			
				C	H	O	N
[$C_{13}H_8O_4$]	Coffee brown	228	272	68.40 (68.42)	3.49 (3.53)	28.00 (28.04)	---

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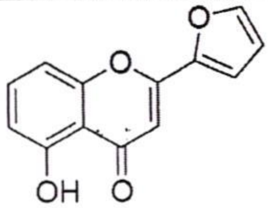

Fig.(2): Chemical structure of $C_{13}H_8O_4$

3.2 Infra red spectrum

The IR spectrum of α, β-unsaturated carbonyl group has characteristic bands of chalcone at prominent bands between 1625 to 1650 per cm.

The characteristic peaks in infra red spectrum give the presence of particular functional group.[9] The region at which other absorption bands appear depends on the type of aromatic / hetero-aromatic rings as well as the substituent present on these rings. The infrared spectrum of chalcone of flavone was recorded on a Perkin- Elmer Spectrum RX-IFTIR Spectrophotometer in the range 4000-400 cm^{-1} using potassium bromide pellet at CIL, Chandigarh, Punjab. The IR bands for 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one appears at 1605 per cm which agrees with the general range of flavone. The change in the position of a band is observed due to change in stretching vibration mode of bond involving coordinated atom. The stretching frequency of flavone is represented in table number (2) and the IR spectrum in figure (3).

Table no. (2): The stretching frequency of flavone

Molecule	υ(OH) Enolic	(-CO-CH=CH-) α,β-unsaturated carbonyl group	Carbonyl group (-C=O in pyron ring)	(C-O-C) Stretching Frequency	(C=C) Stretching Frequency	Aromatic Ring (C=C) Stretching Frequency	Ar-H Stretching Frequency	$-NO_2$ stretching frequency
[$C_{13}H_8O_4$	3516	-	1605	1232	1502	1456	3121	-

Fig.(3): IR spectrum of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one

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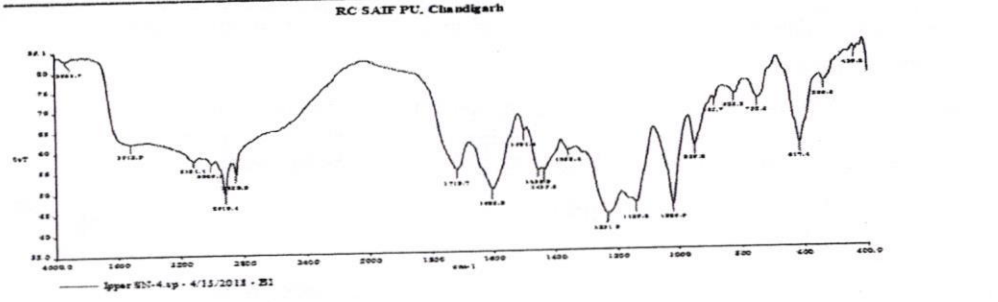

3.3 UV-visible spectrum

Generally, the flavones and flavonols exhibit absorption in 320-380 nm regions (band I) and 240-270 nm regions (band II). The UV-visible spectrum 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one measured on SL 159 single beam UV-VIS spectrophotometer. The UV-visible spectrum of flavone is given in figure (4). In 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one has two matching bands at 245 nm and other at 360 nm which lies in the range of flavone.

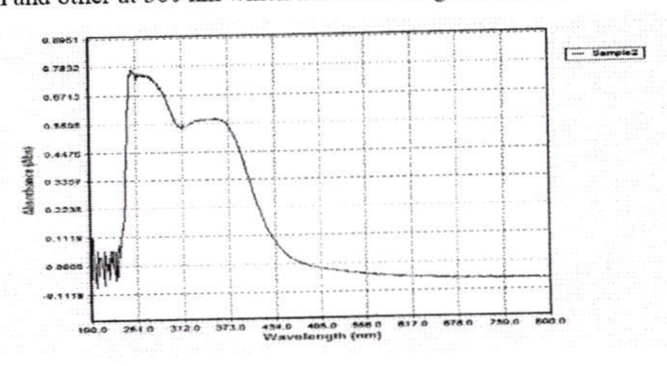

Fig.(4): UV-visible spectrum of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one

3. 4 ^{1}H NMR spectral study

In the analysis of organic molecule, ^{1}H NMR spectra plays very important role. It is the most valuable technique in structural investigation.[10]

The ^{1}H NMR spectrum of chalcone of 5-nitro-furfural is recorded on Bruker AVANCE II 400 MHz Spectrophotometer in DMSO solvent using TMS as an internal standard at SAIF,

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Chandigarh, Punjab are shown in figure (5), and spectral data in table no. (3) and Chemical shift (δ) ppm in figure (5)

Table no. (3): ^{1}H Nuclear magnetic resonance spectral data of chalcone of 5-nitro-furfural

Chemical Shift (δ) ppm	Number of Protons	Multiplicity (Splitting)	Assignment
6.48-7.20	3H	m	Aromatic protons
5.0	1H	s	-OH group present on the aromatic benzene ring
6.71	1H	S	α-H on-unsaturated carbonyl system
6.65-7.80	3H	m	Protons on furan ring

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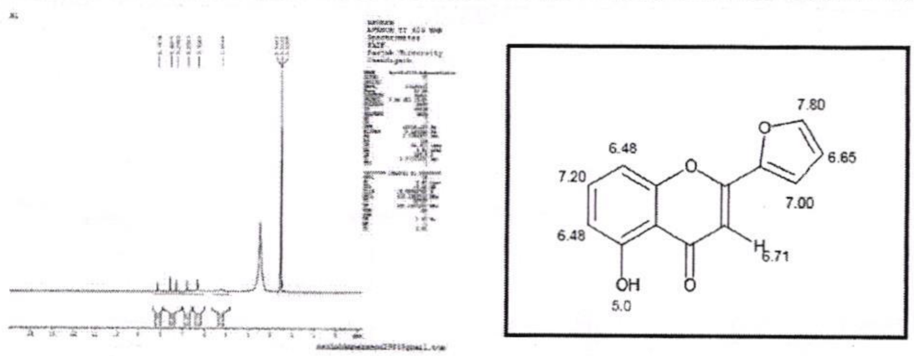

Fig.(5): ^{1}H Nuclear Magnetic Resonance Spectrum of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one and Chemical shift (δ) ppm of protons in PMR spectrum.

3.5 Mass spectrum

Mass spectroscopy is the most accurate technique for the determination of molecular weight of compound. In this technique matter is bombarded with highly energetic electrons. Then matter absorbs or ejects electrons, from it. When it ejects electrons charged species are formed. Anionic and cationic radicals are detected by detector. Detector never detects radical and neutral species. The mass spectrum is a plot representing the m/e values of various ions against their relative percent intensity. The highest intense peak in the spectrum is called base peak. The intensity of other peaks is shown relative to the base peak. The peak at extreme right corresponds to the molecular weight of the original molecule. The molecular ion is called parent ion and usually denoted as $[M]^+$ ion. In mass spectrum peaks are also noticed. Mass spectroscopy also separates the isotopes. The mass spectrum 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one was recorded on Waters, Q-TOF Micro Mass (LC-MS).at SAIF, Chandigarh, Punjab.

The mass spectrum of chalcone of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one is represented in figure (6) and molecular mass. In the mass spectrum of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one, molecular ion peak is observed m/z and calculated m/z corresponding to $[M^+]$ peak are in good agreement with their structure.

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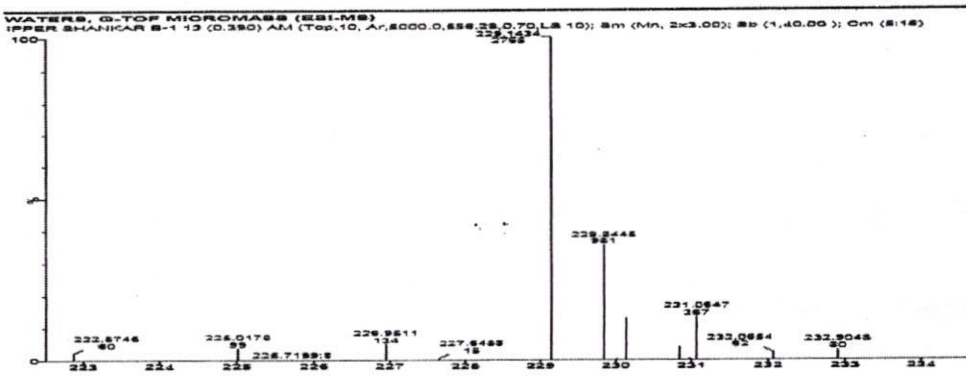

Fig.(6): Mass spectrum of chalcone of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one.

Table no. (4): Antifungal Activity of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one

Ligands	**Antifungal Growth**		
	Aspergillus niger **ATCC 16404**	***Saccharomyces cereviseae*** **ATCC 9763**	***Candida albicans*** **ATCC10231**
	1%	1%	1%
Flavone	12	13	12
Standard	**26**	**24**	**34**

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Table no. (5): Antibacterial Activity of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one

Ligands	Antibacterial Growth Diameter of inhibition zone (mm)									
	S.typhi ATCC 9207	*E.aerogenes* ATCC13048	*B.subtilis* ATCC 6633	*P.aerogenosa* ATCC9027	*S.abony* NCTC 6017	*B.megaterium* ATCC 2326	*E.Coli* ATCC 8739	*S.aureus* ATCC 6538	*S.boydii* ATCC 12034	*B.cereus* ATCC 14579
	1%	1%	1%	1%	1%	1%	1%	1%	1%	1%
Flavone	08	12	10	14	13	14	15	14	13	14
Std	27	33	34	33	30	32	29	25	27	33

The flavone was screened *in vitro* for antifungal activity. The fungal toxicity of flavone was studied *in vitro* against *Aspergillus niger* ATCC 16404, *Saccharomyces cerevisiae* ATCC 9763, *Candida albicans* ATCC10231 fungal pathogens at fixed 1% concentration.

The flavone and was screened *in vitro* for antibacterial activity. The antibacterial activity of flavone was studied, for evaluating antibacterial activity Gram positive and Gram negative bacterial pathogens were used. *Staphylococcus aureus* ATCC 6538, *Bacillus megaterium* ATCC 2326, *Bacillus subtilis* ATCC 6633 were Gram positive pathogens used in this study. *Escherichia coli* ATCC8739, *Salmonella typhi* ATCC9207, *Shigella boydii* ATCC 12034, *Enterobacter aerogenes* ATCC13048, *Pseudomonas aerogenosa* ATCC9027, *Salmonella abony* NCTC6017 were the Gram-negative pathogens used in this study.[11-13]

4. Conclusion

The chalcone of 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one was synthesized by Claisen-Schmidt condensation method. The IUPAC name of this compound is 2-(furan-2-yl)-5-hydroxy-4H-chromen-4-one.The Chemical shift (δ) ppm from ^1H NMR spectral study is between 5.0 - 7.86. The mass spectrum of this chalcone shows molecular ion peak observed m/z and calculated m/z for [M^+], peak is in good agreement. The elemental analysis gives percentage of CHO in chalcone of 5-nitro-furfural.

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Fixed point Functional Theorems differentiability for topological structures with application to Approximation Theory

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Abstract

The purpose of this paper is to discuss the Fixed point theorems differentiability for topological structure. There are many settings in which the fixed point theorems have been studied. Metric spaces and its various generalizations, 2-metric spaces, uniform and quasi-uniform spaces, Banach spaces, normed spaces, 2-normed spaces, locally convex spaces, uniformly convex spaces etc. are some of the settings in which fixed point theorems have been proved and their useful applications have been found.

Key words: *Topological structures, convex spaces, banach spaces, 2- matrix spaces, normed spaces*

Introduction

The theory of fixed points belongs to topology, a part of mathematics created at the end of nineteenth century. The famous French mathematician H. Poincare (1854-1912) was the founder of the fixed point approach. He had deep insight into its future importance for problems of mathematical analysis and celestial mechanics and took an active part in its development.

Fixed point theory is a rich, interesting and exciting branch of mathematics. It is relatively young but fully developed area of research. Study of the existence of fixed points falls within several domains such as, classical analysis, functional analysis, operator theory, general and algebraic topology.

Fixed points and fixed point theorems have always been a major theoretical tool in fields as widely apart as topology, mathematical economics, game theory, and approximation theory and initial and boundary value problems in ordinary and partial differential equations. Moreover, recently, the usefulness of this concept for applications increased enormously by the development of accurate and efficient techniques for computing fixed points, making fixed point methods a major tool in the arsenal of mathematics.

Fixed point theory is equivalent to best approximation, variation inequality and the maximal elements in mathematical economics. The sequence of iterates of fixed point theory can be applied to find solution of a variation inequality and the best approximation theory.

The theory of fixed points is concerned with the conditions which guarantee that a map $T : X \to X$ of a topological space X into itself. admits one or more fixed points that is, points x in X for which x = Tx. For example, a translation, i.e. the mapping $T(x) = x + a$ for a fixed a, has no fixed point, a rotation of the plane has a single fixed point (the center of rotation), the mapping $x \to x^2$ of R into itself has two fixed points (0 and 1) and the projection $(\xi_1, \xi_2) \to \xi_1$ of R^2 into the ξ_1 - axis has infinitely many fixed points (all points of the ξ_1 - axis).

Existence problems of the type $(T - I)x = 0$ arise frequently in analysis. For example, the problem of solving the equation p(z) = 0, where p is a complex polynomial, is equivalent to find a fixed point of the self maps $z - p(z)$ of C. More generally, if $D : M \to E$ is any operator acting on a subset M of a linear space E, to show that the equation $Du = 0$ $[\text{resp.} u - \lambda Du = 0]$ has a solution, is equivalent to show that the map $y \to y - Dy[resp.\ y \to \lambda Dy]$ has a fixed point.

Methodology

Let A, B, S, T, I and J be self mappings of a complete metric space (X, d) satisfying $AB(X) \subset J(X)$, $ST(X) \subset I(X)$ and for each $x, y \in X$ either

$$d(ABx, STy) \le \alpha_1 \left[\frac{\{d(ABx, Jy)\}^2 + \{d(STy, Ix)\}^2}{d(ABx, Jy) + d(STy, Ix)} \right] + \alpha_2 [d(ABx, Ix) + d(STy, Jy)]$$

$$+ \alpha_3 d(Ix, Jy) + F(\min\{d^2(Ix, Jy), d(Ix, ABx).d(Ix, STy), d(Jy.STy).d(Jy, ABx)\})$$

if $d(ABx, Jy) + d(STy, Ix) \ne 0$, $\alpha_i \ge 0 (i = 1,2,3)$ with at least one α_i non zero and $2\alpha_1 + 2\alpha_2 + \alpha_3 < 1$ or

$$d(ABx, STy) = 0 \quad if \quad d(ABx, Jy) + d(STy, Ix) = 0$$

if either (a) (AB, I) are compatible I or AB is continuous and (ST, J) are weakly compatible or (a') (ST, J) are compatible, J or ST is continuous and (AB, I) are weakly compatible, then AB, ST, I and J have a unique common fixed point. Furthermore if the pairs (A,B), (A,I), (B,I), (S,T), (S,J) and (T,J) are commuting mappings then A,B,S,T,I and J have a unique common fixed point.

Proof :

Let $x_0 \in X$, x_0 be an arbitrary point. Since $AB(X) \subset J(X)$, we can choose a point x_1 in X such that $ABx_0 = Jx_1$. Also since $ST(X) \subset I(X)$ we can find a point x_2 with $STx_1 = Ix_2$. Using this argument repeatedly one can construct a sequence $\{z_n\}$ such that

$z_{2n} = ABx_{2n} = Jx_{2n+1}$, $z_{2n+1} = STx_{2n+1} = Ix_{2n+2}$ for $n = 0, 1, 2, \ldots$

for brevity, let us put

$U_{2n} = d(ABx_{2n}, STx_{2n+1})$ and

$U_{2n+1} = d(STx_{2n+1}, ABx_{2n+1})$ for $n = 0,1,2,\ldots\ldots\ldots$

Now we distinguish two cases:

Case - 1 :

Suppose $U_{2n} + U_{2n+1} \neq 0$ for $n = 0,1,2,\ldots.$ Then on using inequality, we have.

$$U_{2n+1} = d(z_{2n+1}, z_{2n+2}) = d(STx_{2n+1}, ABx_{2n+2})$$

$$\leq \alpha_1 \left[\frac{\{d(ABx_{2n+2}, Jx_{2n+1})\}^2 + \{d(STx_{2n+1}, Ix_{2n+2})\}^2}{d(ABx_{2n+2}, Jx_{2n+1}) + d(STx_{2n+1}, Ix_{2n+2})} \right]$$

$$+ \alpha_2 \left[d(ABx_{2n+2}, Ix_{2n+2}) + d(STx_{2n+1}, Jx_{2n+1}) \right] + \alpha_3 d(Ix_{2n+2}, Ix_{2n+1})$$

$$+ F\left(\min\left\{d^2(Ix_{2n+2}, Jx_{2n+1}), d(Ix_{2n+2}, ABx_{2n+2}).d(Ix_{2n+2}, STx_{2n+1}),\right.\right.$$

$$\left.\left. d(Jx_{2n+1}, STx_{2n+1}).d(Jx_{2n+1}, ABx_{2n+2})\right\}\right)$$

$$d(z_{2n+1}, z_{2n+2})$$

$$\leq \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) + \frac{1}{(1 - \alpha_1 + \alpha_2)} F\left(\min\left\{d^2(z_{2n+1}, z_{2n}),\right.\right.$$

$$\left.\left. d(z_{2n+1}, z_{2n+2}).d(z_{2n+1}, z_{2n+1}), d(z_{2n}, z_{2n+1}).d(z_{2n}, z_{2n+2})\right\}\right)$$

$$= \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) + \frac{1}{(1 - \alpha_1 + \alpha_2)} F\left(\min\left\{d^2(z_{2n+1}, z_{2n}),\right.\right.$$

$$\left.\left. d(z_{2n+1}, z_{2n+2}).0, d(z_{2n}, z_{2n+1}).d(z_{2n}, z_{2n+2})\right\}\right)$$

$$= \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) + \frac{1}{(1 - \alpha_1 + \alpha_2)} F\left(\min\left\{d^2(z_{2n+1}, z_{2n}), 0,\right.\right.$$

$$\left.\left. d(z_{2n}, z_{2n+1}).d(z_{2n}, z_{2n+2})\right\}\right)$$

$$= \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) + \frac{1}{(1 - \alpha_1 + \alpha_2)} F(0)$$

$$d(z_{2n+1}, z_{2n+2}) \leq \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) + 0$$

which implies that

$$d(z_{2n+1}, z_{2n+2}) \leq \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1})$$

In the same process one can show that

$$d(z_{2n}, z_{2n+1}) \leq \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n-1}, z_{2n})$$

Thus for every n we have

$$d(z_n, z_{n+1}) \leq kd(z_{n-1}, z_n),$$

where

$$k = \frac{\alpha_1 + \alpha_2 + \alpha_3}{1 - \alpha_1 - \alpha_2} < 1$$

Therefore, by Lemma 2.3.1, $\{z_n\}$ converges to some $z \in X$.

Hence the sequence $ABx_{2n} = Jx_{2n+1}$ and $STx_{2n+1} = Ix_{2n+2}$ which are subsequences also converge to a point z.

Let us now suppose that I is continuous so that the sequence $\{I^2x_{2n}\}$ and $\{IABx_{2n}\}$ converge to the same point Iz.

Since $\{AB, I\}$ are compatible, so the sequence $\{ABIx_{2n}\}$ also converges to the point Iz. We now have,

$$d(ABIx_{2n}, STx_{2n+1})$$
$$\leq \alpha_1 \left[\frac{\{d(ABIx_{2n}, Jx_{2n+1})\}^2 + \{d(STx_{2n+1}, I^2x_{2n})\}^2}{d(ABIx_{2n}, Jx_{2n+1}) + d(STx_{2n+1}, I^2x_{2n})} \right]$$
$$+ \alpha_2 [d(ABIx_{2n}, I^2x_{2n}) + d(STx_{2n+1}, Jx_{2n+1})] + \alpha_3 d(Ix_{2n}, Jx_{2n+1})$$
$$+ F(\min\{d^2(I^2x_{2n}, Jx_{2n+1}), d(I^2x_{2n}, ABx_{2n}).d(I^2x_{2n}, STx_{2n+1}),$$
$$d(Jx_{2n+1}, STx_{2n+1}).d(Jx_{2n+1}, ABIx_{2n})\}),$$

which on letting $n \to \infty$ reduce to

$$d(Iz, z)$$
$$\leq \alpha_1 \left[\frac{\{d(Iz, z)\}^2 + \{d(z, Iz)\}^2}{d(Iz, z) + d(z, Iz)} \right] + \alpha_2 [d(Iz, Iz) + d(z, z)] + \alpha_3 d(Iz, z)$$
$$+ F(\min\{d^2(Iz, z), d(Iz, Iz).d(Iz, z), d(z, z).d(z, Iz)\})$$
$$= (\alpha_1 + \alpha_2) d(z, Iz) + F(\min\{d^2(Iz, z), 0, 0\})$$
$$= (\alpha_1 + \alpha_3) d(z, Iz) + F(0)$$

or $\quad d(Iz, z) \leq (\alpha_1 + \alpha_3) d(z, Iz),$

yielding there by $Iz = z$

Now

$$d(ABz, STx_{2n+1})$$

$$\le \alpha_1\left[\frac{\{d(ABz, Jx_{2n+1})\}^2 + \{d(STx_{2n+1}, Iz)\}^2}{d(ABz, Jx_{2n+1}) + d(STx_{2n+1}, Iz)}\right]$$

$$+\alpha_2[d(ABz, Iz) + d(STx_{2n+1}, Jx_{2n+1})] + \alpha_3 d(Iz, Jx_{2n+1})$$

$$+F(\min\{d^2(Iz, Jx_{2n+1}), d(Iz, ABz).d(Iz, STx_{2n+1}),$$

$$d(Jx_{2n+1}, STx_{2n+1}).d(Jx_{2n+1}, ABz)\})$$

on letting $n \to \infty$ and using $Iz = z$, we get

$$d(ABz, z)$$

$$\le \alpha_1\left[\frac{\{d(ABz, z)\}^2 + \{d(z,z)\}^2}{d(ABz, z) + d(z,z)}\right] + \alpha_2[d(ABz, z) + d(z,z)] + \alpha_3 d(z,z)$$

$$+F(\min\{d^2(z,z), d(z, ABz).d(z,z), d(z,z).d(z, ABz))\})$$

$$\le (\alpha_1 + \alpha_2)d(ABz, z) + F(\min\{0,0,0\})$$

or

$$d(ABz, z) \le (\alpha_1 + \alpha_2)d(ABz, z)$$

implying there by $ABz = z$.

Since z is in the range of AB and $AB(X) \subset J(X)$ there always exists a point z' such that $Jz' = z$ so that $STz = ST(Jz')$

Now

$$d(z, STz') = d(ABz, STz')$$

$$\le \alpha_1\left[\frac{\{d(ABz, Jz')\}^2 + \{d(STz', Iz)\}^2}{d(ABz, Jz') + d(STz', Iz)}\right] + \alpha_2[d(ABz, Iz) + d(STz', Jz')]$$

$$+\alpha_3 d(Iz, Jz') + F(\min\{d^2(Iz, Jz'), d(Iz, ABz).d(Iz, STz'),$$

$$d(Jz', STz').d(Jz', ABz)\})$$

$$= \alpha_1\left[\frac{\{d(z,z)\}^2 + \{d(STz', z)\}^2}{d(z,z) + d(STz, z)}\right] + \alpha_2[d(z,z) + d(STz', z')] + \alpha_3 d(z,z)$$

$$+F(\min\{d^2(z,z), d(z,z).d(z, STz'), d(z, STz').d(z,z)\}),$$

which implies that

$$d(z, STz') \le (\alpha_1 + \alpha_2)d(STz', z)$$

Hence $STz' = z = Jz'$,

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which shows that z' is the coincidence point of ST and J. Now using the weak compatibility of (ST, J) we have

$$STz = ST(Jz') = J(STz') = Jz$$

which shows that z is also a coincidence point of the pair (ST, J). Now

$$\begin{aligned} d(z,STz) &= d(ABz,STz) \\ &\le \alpha_1\left[\frac{\{d(ABz,Jz)\}^2+\{d(STz,Iz)\}^2}{d(ABz,Jz)+d(STz,Iz)}\right]+\alpha_2[d(ABz,Iz)+d(STz,Jz)] \\ &\quad +\alpha_3 d(Iz,Jz)+F(\min\{d^2(Iz,Jz'),d(Iz,ABz).d(Iz,STz), \\ &\quad d(Jz,STz).d(Jz,ABz)\}) \end{aligned}$$

$$\begin{aligned} &= \alpha_1\left[\frac{\{d(z,STz)\}^2+\{d(STz,z)\}^2}{d(z,STz)+d(STz,Iz)}\right]+\alpha_2[d(z,z)+d(STz,STz)] \\ &+\alpha_3 d(z,STz)+F(\min\{d^2(z,STz),d(z,z).d(z,STz), \\ &\quad d(z,STz).d(STz,z)\}) \\ &= (\alpha_1+\alpha_2)d(z,STz)+F(\min\{d^2(z,STz),0,0\}) \\ &= (\alpha_1+\alpha_3)d(z,STz)+F(0), \end{aligned}$$

Which implies that

$$(d,STz)\le(\alpha_1+\alpha_3)d(z,STz)$$

Hence $z = STz = Jz$ which shows that z is a common fixed point of AB, I, ST and J.

Now suppose that AB is continuous so that the sequences $\{AB^2x_{2n}\}$ and $\{ABIx_{2n}\}$ converges to ABz. Since (AB, I) are compatible it follows that $\{IABx_{2n}\}$ also converges to ABz. Thus

$$\begin{aligned} &d(AB^2x_{2n},STx_{2n+1}) \\ &\le \alpha_1\left[\frac{\{d(AB^2x_{2n},Jx_{2n+1})\}^2+\{d(STx_{2n+1},IABx_{2n})\}^2}{d(AB^2x_{2n},Jx_{2n+1})+d(STx_{2n+1},IABx_{2n})}\right] \\ &+\alpha_2[d(AB^2x_{2n},IABx_{2n})+d(STx_{2n+1},Jx_{2n+1})]+\alpha_3 d(IABx_{2n},Jx_{2n+1}) \\ &+F(\min\{d^2(IABx_{2n},Jx_{2n+1}),d(IABx_{2n},AB^2x_{2n}).d(IABx_{2n},STx_{2n+1}), \\ &\quad d(Jx_{2n+1},STx_{2n+1}).d(Jx_{2n+1},ABIx_{2n})\}), \end{aligned}$$

which on letting $n\to\infty$ reduces to

$d(ABz, z)$

$$\leq \alpha_1\left[\frac{\{d(ABz,z)\}^2+\{d(z,ABz)\}^2}{d(ABz,z)+d(z,ABz)}\right]+\alpha_2[d(ABz,ABz)+d(z,z)]$$

$$+\alpha_3 d(ABz,z)+F(\min\{d^2(ABz,z),d(ABz,ABz).d(ABz,z),$$

$$d(z,z).d(z,ABz)\})$$

$$=(\alpha_1+\alpha_2)d(ABz,z)+F(\min\{d^2(ABz,z).0,0\})$$

or

$$d(ABz,z)\leq(\alpha_1+\alpha_3)d(ABz,z)$$

Yielding thereby $ABz = z$

As earlier there exists z' in X such that $ABz = z = Jz'$. Then

$$d(AB^2x_{2n},STz')$$

$$\leq \alpha_1\left[\frac{\{d(AB^2x_{2n},Jz')\}^2+\{d(STz',IABx_{2n})\}^2}{d(AB^2x_{2n},Jz')+d(STz',IABx_{2n})}\right]+\alpha_2[d(AB^2x_{2n},IABx_{2n})$$

$$+d(STz',Jz')]+\alpha_3 d(IABx_{2n},Jz')+F(\min\{d^2(IABx_{2n},Jz'),$$

$$d(IABx_{2n},AB^2x_{2n}).d(IABx_{2n},STz'),d(Jz',STz').d(Jz',AB^2x_{2n})\})$$

which on letting $n\to\infty$, reduces to

$$d(z.STz')$$

$$\leq(\alpha_1+\alpha_2)d(z,STz')+F(\min\{d^2(ABz,z),d(ABz,ABz).d(ABz,STz'),$$

$$d(ABz,STz')d(ABz,ABz)\})$$

or $d(z,STz')\leq(\alpha_1+\alpha_2)d(z,STz')+F(\min\{d^2(ABz,z),0,0\})$

or $d(z,STz')\leq(\alpha_1+\alpha_3)d(z,STz')$,

from this we obtain $STz' = z = Jz'$. Thus the pair (*ST,J*) has a coincidence point z'. Since the pair (*ST, J*) is weakly compatible then we have

$STz = ST(Jz') = J(ST\ z') = Jz$, which shows that $STz = Jz$

Further.

$$d(ABx_{2n},STz)$$

$$\leq \alpha_1\left[\frac{\left\{d\left(ABx_{2n}, Jz\right)^2\right\}+\left\{d\left(STz, Ix_{2n}\right)^2\right\}}{d\left(ABx_{2n}, Jz\right)+d\left(STz, Ix_{2n}\right)}\right]+\alpha_2\left[d\left(ABx_{2n}, Ix_{2n}\right)\right.$$

$$\left.+d\left(STz, Jz\right)\right]+\alpha_3 d\left(Ix_{2n}, Jz\right)+F\left(\min\left\{d^2\left(Ix_{2n}, Jz\right), d\left(Ix_{2n}, ABx_{2n}\right).\right.\right.$$

$$\left.\left.d\left(Ix_{2n}, STz\right), d\left(Jz, STz\right).d\left(Jz, ABx_{2n}\right)\right\}\right),$$

which on letting $n \to \infty$, we get

$$d\left(z.STz\right)$$

$$\leq \alpha_1\left[\frac{\left\{d\left(z, STz\right)\right\}^2+\left\{d\left(STz, z\right)\right\}^2}{d(z, STz)+d(STz, z)}\right]+\alpha_2\left[d(z,z)+d(STz, STz)\right]$$

$$+F\left(\min\left\{d^2\left(z, STz\right), d\left(z,z\right)d\left(z, STz\right), d\left(STz, STz\right)d\left(STz, z\right)\right\}\right)$$

$$=\left(\alpha_1+\alpha_2\right)d\left(z, STz\right)+F\left(\min\left\{d^2(z, STz), 0, 0\right\}\right)$$

or, equivalently

$$d(z, STz) \leq \left(\alpha_1+\alpha_3\right) d\left(z, STz\right),$$

which implies that $STz = z = Jz$

Since $ST(X) \subset I(X)$ and $STz = z$ there exist a point z'' in X such that $Iz'' = z$. Thus

$$d\left(ABz'', z\right)=d\left(ABz'', STz\right)$$

$$\leq \alpha_1\left[\frac{\left\{d\left(ABz'', Jz\right)\right\}^2+\left\{d\left(Iz''\right)\right\}^2}{d(ABz'', Jz)+d(STz, Iz'')}\right]+\alpha_2\left[d(ABz'', Iz'')+d(STz, Jz)\right]$$

$$+\alpha_2 d(Iz'', Jz)\left[d+d(STz, Jz)\right]+F\left(\min\left\{d^2\left(Iz'', Jz\right), d\left(Iz'', ABz''\right).\right.\right.$$

$$\left.\left.d\left(Iz'', STz\right), d\left(Jz, STz\right).d\left(Jz, ABz''\right)\right\}\right)$$

or $d\left(ABz'', z\right)$

$$\leq \left(\alpha_1+\alpha_2\right)d\left(ABz'', z\right)+F\left(\min\left\{0, d\left(z, ABz''\right).d\left(z, STz\right), d\left(z,z\right).d\left(z, ABz''\right)\right\}\right)$$

or $d\left(ABz'', z\right) \leq \left(\alpha_1+\alpha_3\right) d\left(ABz'', z\right)$

which shows that $ABz'' = z.$

Also since (AB, I) are compatible and hence weakly commuting we obtain.

$$d\left(ABz, Iz\right)=d\left(AB\left(Iz''\right), I\left(ABz''\right)\right)$$

$$\leq d\left(Iz'', ABz''\right)=d\left(z,z\right)=0$$

Therefore $ABz = Iz = z$.

Thus we have proved that z is a common fixed point of AB, ST, I and J.

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Conclusion

Fixed point theory is equivalent to best approximation, variation inequality and the maximal elements in mathematical economics. The sequence of iterates of fixed point theory can be applied to find solution of a variation inequality and the best approximation theory.

The conclusion of the present paper is that Common fixed point theorems for asymptotically commenting mappings and certain composite involution in Banach spaces have been obtained.

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Fine Structure Operators For Mappings In Metric Spaces

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Abstract

In this paper, some common Fine Structure theorems for six mappings involving rational inequalities in complete metric-spaces have been proved by using the notions of compatibility and commutatively. Using the concept of relative asymptotic regularity at a point for compatible and weakly commuting mappings, some common Fine Structure theorems in complete 2-metric spaces have been proved also new notions of composite asymptotic regular mapping have been defined and to use the concept for proving some common Fine Structure theorems.

Keywords: Fine Structure, mappings, rational inequalities, complete metric-spaces, weakly commuting mapings, compatibility, 2-metric spaces ,composite asymptotic regular mapping.

Introduction:

The theory of Fine Structure is one of the most important and powerful tools of the modern mathematics not only it is used on a daily bases in pure and applied mathematics but it is also solves a bridge between analysis and topology and provide a very fruitful are of interaction between the two.

1.1 FINE STRUCTURE THEORY :

Fine Structure theory is a rich, interesting and exciting branch of mathematics. It is relatively young but fully developed area of research. Study of the existence of Fine Structures falls within several domains such as, classical analysis, functional analysis, operator theory, general and algebraic topology.

Fine Structures and Fine Structure theorems have always been a major theoretical tool in fields as widely apart as topology, mathematical economics, game theory, and approximation theory and initial and boundary value problems in ordinary and partial differential equations. Moreover, recently, the usefulness of this concept for applications increased enormously by the development of accurate and efficient techniques for computing Fine Structures, making Fine Structure methods a major tool in the arsenal of mathematics.

2.2.1 Definition:

Let S and T be mappings of a metric space (X, d) into itself. Then (S, T) is said to be **weakly commuting** pair if

$d(STx, TSx) \le d(Tx, Sx)$ for all $x \in X$.

Obviously a commuting pair is weakly commuting but its converse need not be true as is evident from the following example.

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2.2.2 Example:

Consider the set $X = [0, 1]$ with the usual metric. Let $Sx = \dfrac{x}{2}$ and $Tx = \dfrac{x}{2+x}$ for every $x \in X$. Then for all $x \in X$

$$STx = \frac{x}{4+2x}, \quad TSx = \frac{x}{4+x}$$

hence $ST \neq TS$. Thus S and T do not commute.

2.2.3 Definition:

Two self mappings S and T of a metric space (X, d) are **compatible** if and only if.

$\lim_{n\to\infty} d(STx_n, TSx_n) = 0$ whenever $\{x_n\}$ is a sequence in X. such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

Clearly any weakly commuting pair $\{S,T\}$ in compatible but the converse need not be true as can be seen in the following example.

2.2.4 Example:

Let $Sx = x^3$ and $Tx = 2x^3$ with $X = R$ with the usual metric. Then S and T are compatible, since

$$|Tx - Sx| = |x^3| \to 0 \text{ if and only if}$$

$$|STx - TSx| = 6|x^9| \to 0 \text{ but}$$

$|STx - TSx| \leq |Tx - Sx|$ is not true for all $x \in X$, say for example at $x = 1$.

More, recently, Jungck et. al. [10] introduced the concept of compatible mapping of type (A) which is stated as follows:

2.2.5 Definition:

Let S and T be mappings from a metric space (X, d) into itself. The pair (S, T) is said to be **compatible of type (*A*)** on X if whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z \text{ in } X \text{ then}$$

$$d(STx_n, TTx_n) \to 0 \text{ and } d(TSx_n, SSx_n) \to 0 \text{ as } n \to \infty$$

It is shown in [10] that under certain conditions the compatible and compatible (type A) mappings are equivalent for instance.

2.2.6 Proposition:

Let *S* and *T* be continuous self mapping on *X*. Then the pair (*S, T*) is compatible on X. where as in (Jungck [10], Gajic [4]) demonstrated by suitable examples that if *S* and *T* are discontinuous then the two concepts are independent of each other. The following examples also support this observation.

2.2.7 Example:

Let $X = R$ with the usual metric we define $S, T : X \to X$ as follows.

$$Sx = \begin{cases} 1/x^2 & x \neq 0 \\ 0 & x = 0 \end{cases} \quad and \quad Tx = \begin{cases} 1/x^3 & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Both *S* and *T* are discontinuous at $x = 0$ and for any sequence $\{x_n\}$ in *X*, we have $d(STx_n, TSx_n) = 0$. Hence the pair (S,T) is compatible. Now consider the sequence $x_n = n \in N$. Then $Sx_n \to 0$ and $Tx_n \to 0$ as $n \to \infty$ and

$$d(STx_n, TTx_n) = |x^6 - x^9| \to \infty \quad as\ n \to \infty$$

Hence the pair (*S,T*) is not compatible of type (*A*)

2.2.8 Example:

Now we define

$$Sx = \begin{cases} 1/x^3, & x > 1 \\ 1, & 0 \leq x \leq 1 \\ 0, & x < 0 \end{cases} \quad and \quad Tx = \begin{cases} -1/x^3, & x > 1 \\ 1, & 0 \leq x \leq 1 \\ 0, & x < 0 \end{cases}$$

observe that the restriction of *S* and *T* on $(-\infty, 1]$ are equal, thus we take a sequence $\{x_n\}$ in $(1, \infty)$. Then $\{Sx_n\} \subset (0,1)$ *and* $\{Tx_n\} \subset (-1,0)$. Thus for every *n*, $TTx_n = 0$, $TSx_n = 1$, $STx_n = 0$, $SSx_n = 1$. So that $d(STx_n, TTx_n) = 0$, $d(TSx_n, TTx_n) = 0$ for every $n \in N$. This shows that the pair (*S, T*) is compatible of type (A).

Now let $x_n = n,\ n \in N$. Then $Tx_n \to 0, Sx_n \to 0\ as\ n \to \infty\ and\ STx_n = 0, TSx_n = 1$ for every $n \in N$ and so $d(STx_n, TSx_n) \neq 0$ as $n \to \infty$ hence the pair (S, T) is not compatible.

Very recently concept of **weakly compatible** obtained by Jungck-Rhoades [7] stated as the pair of mappings is said to be weakly compatible if they commute at their coincidence point.

2.2.9 Example :

Let X = [2, 20] with usual metric define

$$Tx = \begin{cases} 2 & if\ x = 2 \\ 12 + x & if\ \ 2 < x \le 5 \\ x - 3 & if\ \ 5 < x \le 20 \end{cases} \quad and \quad Sx = \begin{cases} 2 & if\ \ x \in \{2\} \cup (5,20] \\ 8 & if\ \ 2 < x \le 5 \end{cases}$$

S and *T* are weakly compatible mappings which is not compatible. To see that *S* and *T* are not compatible of Type (A). Let us consider a decreasing sequence $\{x_n\}$ such that

$$\lim x_n = 5$$

Then

$$Tx_n = x_n - 3 \to 2;\ Sx_n = 2;\ STx_n = S(x_n - 3) = 8\ and$$

$$TTxn = T(x_n - 3) = 12 + x_n - 3 \to 14, that\ is$$

$$\lim d(STx_n, TTx_n) = 6 \ne 0$$

and hence *S* and *T* are not compatible of type (A).

2.3.0 Theorem:

Let *A*, *B*,*S*, *T*, *I* and *J* be self mappings of a complete metric space (*X*, *d*) satisfying $AB\,(X) \subset J\,(X)$, $ST\,(X) \subset I\,(X)$ and for each $x, y \in X$ either

$$d(ABx, STy) \le \alpha_1 \left[\frac{\{d(ABx, Jy)\}^2 + \{d(STy, Ix)\}^2}{d(ABx, Jy) + d(STy, Ix)} \right] + \alpha_2 [d(ABx, Ix) + d(STy, Jy)]$$
$$+ \alpha_3 d(Ix, Jy) + F(\min\{d^2(Ix, Jy), d(Ix, ABx).d(Ix, STy), d(Jy.STy).d(Jy, ABx)\}) \quad ....(2.3.1)$$

if $d(ABx, Jy) + d(STy, Ix) \ne 0$, $\alpha_i \ge 0 (i = 1,2,3)$ with at least one α_i non zero and $2\alpha_1 + 2\alpha_2 + \alpha_3 < 1$ or

$$d(ABx, STy) = 0 \quad if \quad d(ABx, Jy) + d(STy, Ix) = 0 \quad ....(2.3.2)$$

if either (a) (*AB*, *I*) are compatible *I* or *AB* is continuous and (*ST*, *J*) are weakly compatible or

(a') (*ST*, *J*) are compatible, *J* or *ST* is continuous and (*AB*, *I*) are weakly compatible, then *AB*, *ST*, *I* and *J* have a unique common Fine Structure. Furthermore if the pairs (*A*,*B*), (*A*,*I*), (*B*,*I*), (*S*,*T*), (*S*,*J*) and (*T*,*J*) are commuting mappings then *A*,*B*,*S*,*T*,*I* and *J* have a unique common Fine Structure.

Proof:

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Let $x_0 \in X$, x_0 be an arbitrary point. Since $AB(X) \subset J(X)$, we can choose a point x_1 in X such that $ABx_0 = JX_1$. Also since $ST(X) \subset I(X)$ we can find a point x_2 with $STx_1=Ix_2$. Using this argument repeatedly one can construct a sequence $\{z_n\}$ such that

$$z_{2n} = ABx_{2n}=Jx_{2n+1},\ z_{2n+1} = STx_{2n+1}= Ix_{2n+2} \text{ for } n = 0, 1, 2,\ldots.$$

for brevity, let us put

$U_{2n} = d(ABx_{2n}, STx_{2n+1})$ and

$U_{2n+1} = d(STx_{2n+1}, ABx_{2n+1})$ for $n = 0,1,2,\ldots\ldots\ldots$

Now we distinguish two cases:

Case - 1 :

Suppose $U_{2n} + U_{2n+1} \neq 0$ for $n = 0,1,2,\ldots.$ Then on using inequality (2.3.1), we have.

$$U_{2n+1} = d(z_{2n+1}, z_{2n+2}) = d(STx_{2n+1}, ABx_{2n+2})$$

$$\leq \alpha_1 \left[\frac{\{d(ABx_{2n+2}, Jx_{2n+1})\}^2 + \{d(STx_{2n+1}, Ix_{2n+2})\}^2}{d(ABx_{2n+2}, Jx_{2n+1}) + d(STx_{2n+1}, Ix_{2n+2})} \right]$$

$$+ \alpha_2 \left[d(ABx_{2n+2}, Ix_{2n+2}) + d(STx_{2n+1}, Jx_{2n+1}) \right] + \alpha_3 d(Ix_{2n+2}, Ix_{2n+1})$$

$$+ F\big(\min\{d^2(Ix_{2n+2}, Jx_{2n+1}), d(Ix_{2n+2}, ABx_{2n+2}).d(Ix_{2n+2}, STx_{2n+1}),$$

$$d(Jx_{2n+1}, STx_{2n+1}).d(Jx_{2n+1}, ABx_{2n+2})\}\big)$$

$$d(z_{2n+1}, z_{2n+2})$$

$$\leq \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) + \frac{1}{(1 - \alpha_1 + \alpha_2)} F\big(\min\{d^2(z_{2n+1}, z_{2n}),$$

$$d(z_{2n+1}, z_{2n+2}).d(z_{2n+1}, z_{2n+1}), d(z_{2n}, z_{2n+1}).d(z_{2n}, z_{2n+2})\}\big)$$

$$= \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) + \frac{1}{(1 - \alpha_1 + \alpha_2)} F\big(\min\{d^2(z_{2n+1}, z_{2n}),$$

$$d(z_{2n+1}, z_{2n+2}).0, d(z_{2n}, z_{2n+1}).d(z_{2n}, z_{2n+2})\}\big)$$

$$= \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) + \frac{1}{(1 - \alpha_1 + \alpha_2)} F\big(\min\{d^2(z_{2n+1}, z_{2n}), 0,$$

$$d(z_{2n}, z_{2n+1}).d(z_{2n}, z_{2n+2})\}\big)$$

$$= \frac{(\alpha_1 + \alpha_2 + \alpha_3)}{(1 - \alpha_1 - \alpha_2)} d(z_{2n}, z_{2n+1}) + \frac{1}{(1 - \alpha_1 + \alpha_2)} F(0)$$

$$d(z_{2n+1}, z_{2n+2}) \le \frac{(\alpha_1+\alpha_2+\alpha_3)}{(1-\alpha_1-\alpha_2)} d(z_{2n}, z_{2n+1}) + 0$$

which implies that

$$d(z_{2n+1}, z_{2n+2}) \le \frac{(\alpha_1+\alpha_2+\alpha_3)}{(1-\alpha_1-\alpha_2)} d(z_{2n}, z_{2n+1})$$

In the same process one can show that

$$d(z_{2n}, z_{2n+1}) \le \frac{(\alpha_1+\alpha_2+\alpha_3)}{(1-\alpha_1-\alpha_2)} d(z_{2n-1}, z_{2n})$$

Thus for every n we have

$$d(z_n, z_{n+1}) \le k d(z_{n-1}, z_n), \qquad ....(2.3.2)$$

where

$$k = \frac{\alpha_1+\alpha_2+\alpha_3}{1-\alpha_1-\alpha_2} < 1$$

Therefore, by Lemma 2.3.1, $\{z_n\}$ converges to some $z \in X$.

Hence the sequence $ABx_{2n} = Jx_{2n+1}$ and $STx_{2n+1} = Ix_{2n+2}$ which are subsequences also converge to a point z.

Let us now suppose that I is continuous so that the sequence $\{I^2x_{2n}\}$ and $\{IABx_{2n}\}$ converge to the same point Iz.

Since $\{AB, I\}$ are compatible, so the sequence $\{ABIx_{2n}\}$ also converges to the point Iz.

We now have,

$$\begin{aligned} d(ABIx_{2n}, STx_{2n+1}) &\le \alpha_1\left[\frac{\{d(ABIx_{2n}, Jx_{2n+1})\}^2 + \{d(STx_{2n+1}, I^2x_{2n})\}^2}{d(ABIx_{2n}, Jx_{2n+1}) + d(STx_{2n+1}, I^2x_{2n})}\right] \\ &+ \alpha_2[d(ABIx_{2n}, I^2x_{2n}) + d(STx_{2n+1}, Jx_{2n+1})] + \alpha_3 d(Ix_{2n}, Jx_{2n+1}) \\ &+ F(\min\{d^2(I^2x_{2n}, Jx_{2n+1}), d(I^2x_{2n}, ABx_{2n}) d(I^2x_{2n}, STx_{2n+1}), \\ &\quad d(Jx_{2n+1}, STx_{2n+1}).d(Jx_{2n+1}, ABIx_{2n})\}), \end{aligned}$$

which on letting $n \to \infty$ reduce to

$$\begin{aligned} d(Iz, z) &\le \alpha_1\left[\frac{\{d(Iz,z)\}^2 + \{d(z,Iz)\}^2}{d(Iz,z) + d(z,Iz)}\right] + \alpha_2[d(Iz,Iz) + d(z,z)] + \alpha_3 d(Iz,z) \\ &+ F(\min\{d^2(Iz,z), d(Iz,Iz).d(Iz,z), d(z,z).d(z,Iz)\}) \end{aligned}$$

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$$= (\alpha_1 + \alpha_2) d(z, Iz) + F(\min\{d^2(Iz, z), 0, 0\})$$
$$= (\alpha_1 + \alpha_3) d(z, Iz) + F(0)$$

or $\quad d(Iz, z) \le (\alpha_1 + \alpha_3) d(z, Iz)$,

yielding thereby $Iz = z$

Now

$$d(ABz, STx_{2n+1})$$
$$\le \alpha_1 \left[\frac{\{d(ABz, Jx_{2n+1})\}^2 + \{d(STx_{2n+1}, Iz)\}^2}{d(ABz, Jx_{2n+1}) + d(STx_{2n+1}, Iz)} \right]$$
$$+ \alpha_2 [d(ABz, Iz) + d(STx_{2n+1}, Jx_{2n+1})] + \alpha_3 d(Iz, Jx_{2n+1})$$
$$+ F(\min\{d^2(Iz, Jx_{2n+1}), d(Iz, ABz).d(Iz, STx_{2n+1}),$$
$$d(Jx_{2n+1}, STx_{2n+1}).d(Jx_{2n+1}, ABz)\})$$

on letting $n \to \infty$ and using $Iz = z$, we get

$$d(ABz, z)$$
$$\le \alpha_1 \left[\frac{\{d(ABz, z)\}^2 + \{d(z, z)\}^2}{d(ABz, z) + d(z, z)} \right] + \alpha_2 [d(ABz, z) + d(z, z)] + \alpha_3 d(z, z)$$
$$+ F(\min\{d^2(z, z), d(z, ABz).d(z, z), d(z, z).d(z, ABz))\})$$
$$\le (\alpha_1 + \alpha_2) d(ABz, z) + F(\min\{0, 0, 0\})$$

or

$$d(ABz, z) \le (\alpha_1 + \alpha_2) d(ABz, z)$$

implying thereby $ABz = z$.

Since z is in the range of AB and $AB(X) \subset J(X)$ there always exists a point z' such that $Jz' = z$ so that $STz = ST(Jz')$

Now

$$d(z, STz') = d(ABz, STz')$$

$$\le \alpha_1\left[\frac{\{d(ABz,Jz')\}^2+\{d(STz',Iz)\}^2}{d(ABz,Jz')+d(STz',Iz)}\right]+\alpha_2[d(ABz,Iz)+d(STz',Jz')]$$

$$+\alpha_3 d(Iz,Jz')+F(\min\{d^2(Iz,Jz'),d(Iz,ABz)d(Iz,STz'),$$

$$d(Jz',STz').d(Jz',ABz)\})$$

$$=\alpha_1\left[\frac{\{d(z,z)\}^2+\{d(STz',z)\}^2}{d(z,z)+d(STz,z)}\right]+\alpha_2[d(z,z)+d(STz',z')]+\alpha_3 d(z,z)$$

$$+F(\min\{d^2(z,z),d(z,z).d(z,STz'),d(z,STz')d(z,z)\}),$$

which implies that

$$d(z,STz')\le(\alpha_1+\alpha_2)d(STz',z)$$

Hence $STz'=z=Jz'$,
which shows that z' is the coincidence point of *ST* and *J*. Now using the weak compatibility of (*ST*, *J*) we have

$$STz=ST(Jz')=J(STz')=Jz$$

which shows that z is also a coincidence point of the pair (*ST*, *J*). Now

$$d(z,STz)=d(ABz,STz)$$

$$\le\alpha_1\left[\frac{\{d(ABz,Jz)\}^2+\{d(STz,Iz)\}^2}{d(ABz,Jz)+d(STz,Iz)}\right]+\alpha_2[d(ABz,Iz)+d(STz,Jz)]$$

$$+\alpha_3 d(Iz,Jz)+F(\min\{d^2(Iz,Jz'),d(Iz,ABz).d(Iz,STz),$$

$$d(Jz,STz).d(Jz,ABz)\})$$

$$=\alpha_1\left[\frac{\{d(z,STz)\}^2+\{d(STz,z)\}^2}{d(z,STz)+d(STz,Iz)}\right]+\alpha_2[d(z,z)+d(STz,STz)]$$

$$+\alpha_3 d(z,STz)+F(\min\{d^2(z,STz),d(z,z)d(z,STz),$$

$$d(z,STz)d(STz,z)\})$$

$$=(\alpha_1+\alpha_2)d(z,STz)+F(\min\{d^2(z,STz),0,0\})$$

$$=(\alpha_1+\alpha_3)d(z,STz)+F(0),$$

which implies that

$$(d,STz)\le(\alpha_1+\alpha_3)d(z,STz)$$

Hence $z=STz=Jz$ which shows that z is a common Fine Structure of *AB*, *I*, *ST* and *J*.

Now suppose that AB is continuous so that the sequences $\{AB^2x_{2n}\}$ and $\{ABIx_{2n}\}$ converges to ABz. Since (AB, I) are compatible it follows that $\{IABx_{2n}\}$ also converges to ABz. Thus

$$d\left(AB^2x_{2n}, STx_{2n+1}\right)$$
$$\leq \alpha_1\left[\frac{\{d(AB^2x_{2n}, Jx_{2n+1})\}^2 + \{d(STx_{2n+1}, IABx_{2n})\}^2}{d(AB^2x_{2n}, Jx_{2n+1}) + d(STx_{2n+1}, IABx_{2n})}\right]$$
$$+\alpha_2\left[d\left(AB^2x_{2n}, IABx_{2n}\right) + d\left(STx_{2n+1}, Jx_{2n+1}\right)\right] + \alpha_3 d\left(IABx_{2n}, Jx_{2n+1}\right)$$
$$+F\left(\min\{d^2(IABx_{2n}, Jx_{2n+1}), d(IABx_{2n}, AB^2x_{2n})d(IABx_{2n}, STx_{2n+1}),\right.$$
$$\left. d(Jx_{2n+1}, STx_{2n+1}).d(Jx_{2n+1}, ABIx_{2n})\}\right),$$

which on letting $n \to \infty$ reduces to

$$d(ABz, z)$$
$$\leq \alpha_1\left[\frac{\{d(ABz, z)\}^2 + \{d(z, ABz)\}^2}{d(ABz, z) + d(z, ABz)}\right] + \alpha_2[d(ABz, ABz) + d(z, z)]$$
$$+\alpha_3 d(ABz, z) + F\left(\min\{d^2(ABz, z), d(ABz, ABz).d(ABz, z),\right.$$
$$\left. d(z, z).d(z, ABz)\}\right)$$
$$= (\alpha_1 + \alpha_2)d(ABz, z) + F\left(\min\{d^2(ABz, z).0, 0\}\right)$$

or

$$d(ABz, z) \leq (\alpha_1 + \alpha_3)d(ABz, z)$$

yielding thereby $ABz = z$

As earlier there exists z' in X such that $ABz = z = Jz'$. Then

$$d\left(AB^2x_{2n}, STz'\right)$$
$$\leq \alpha_1\left[\frac{\{d(AB^2x_{2n}, Jz')\}^2 + \{d(STz', IABx_{2n})\}^2}{d(AB^2x_{2n}, Jz') + d(STz', IABx_{2n})}\right] + \alpha_2\left[d\left(AB^2x_{2n}, IABx_{2n}\right)\right.$$
$$\left.+ d(STz', Jz')\right] + \alpha_3 d\left(IABx_{2n}, Jz'\right) + F\left(\min\{d^2(IABx_{2n}, Jz'),\right.$$
$$\left. d(IABx_{2n}, AB^2x_{2n}).d(IABx_{2n}, STz'), d(Jz', STz').d(Jz', AB^2x_{2n})\}\right)$$

which on letting $n \to \infty$, reduces to

$$d(z.STz')$$
$$\leq (\alpha_1 + \alpha_2)d(z, STz') + F\left(\min\{d^2(ABz, z), d(ABz, ABz).d(ABz, STz'),\right.$$
$$\left. d(ABz, STz')d(ABz, ABz)\}\right)$$

or $d(z, STz') \le (\alpha_1 + \alpha_2) d(z, STz') + F(\min\{d^2(ABz, z), 0, 0\})$

or $d(z, STz') \le (\alpha_1 + \alpha_3) d(z, STz')$,

from this we obtain $STz' = z = Jz'$. Thus the pair (ST, J) has a coincidence point z'. Since the pair (ST, J) is weakly compatible then we have

$STz = ST(Jz') = J(ST\ z') = Jz$, which shows that $STz = Jz$

Further.

$$d(ABx_{2n}, STz) \le \alpha_1 \left[\frac{\{d(ABx_{2n}, Jz)^2\} + \{d(STz, Ix_{2n})^2\}}{d(ABx_{2n}, Jz) + d(STz, Ix_{2n})} \right] + \alpha_2 [d(ABx_{2n}, Ix_{2n})$$

$$+ d(STz, Jz)] + \alpha_3 d(Ix_{2n}, Jz) + F(\min\{d^2(Ix_{2n}, Jz), d(Ix_{2n}, ABx_{2n}).$$

$$d(Ix_{2n}, STz), d(Jz, STz).d(Jz, ABx_{2n})\}),$$

which on letting $n \to \infty$, we get

$$d(z.STz) \le \alpha_1 \left[\frac{\{d(z, STz)\}^2 + \{d(STz, z)\}^2}{d(z, STz) + d(STz, z)} \right] + \alpha_2 [d(z, z) + d(STz, STz)]$$

$$+ F(\min\{d^2(z, STz), d(z, z) d(z, STz), d(STz, STz) d(STz, z)\})$$

$$= (\alpha_1 + \alpha_2) d(z, STz) + F(\min\{d^2(z, STz), 0, 0\})$$

or, equivalently

$d(z, STz) \le (\alpha_1 + \alpha_3)\, d(z, STz)$,

which implies that $STz = z = Jz$

Since $ST(X) \subset I(X)$ and $STz = z$ there exist a point z'' in X such that $Iz'' = z$. Thus

$$d(ABz'', z) = d(ABz'', STz) \le \alpha_1 \left[\frac{\{d(ABz'', Jz)\}^2 + \{d(Iz'')\}^2}{d(ABz'', Jz) + d(STz, Iz'')} \right] + \alpha_2 [d(ABz'', Iz'') + d(STz, Jz)]$$

$$+ \alpha_2 d(Iz'', Jz)[d + d(STz, Jz)] + F(\min\{d^2(Iz'', Jz), d(Iz'', ABz'').$$

$$d(Iz'', STz), d(Jz, STz).d(Jz, ABz'')\})$$

or $d(ABz'', z)$

$$\le (\alpha_1 + \alpha_2) d(ABz'', z) + F(\min\{0, d(z, ABz'').d(z, STz), d(z, z).d(z, ABz'')\})$$

or $d(ABz'', z) \le (\alpha_1 + \alpha_3)\, d(ABz'', z)$

which shows that $ABz'' = z$.

Also since (*AB,I*) are compatible and hence weakly commuting we obtain.

$$d(ABz,Iz) = d(AB(Iz''), I(ABz'))$$
$$\leq d(Iz'', ABz'') = d(z,z) = 0$$

Therefore $ABz = Iz = z$.

Thus we have proved that *z* is a common Fine Structure of *AB*, *ST*, *I* and *J*.

Instead of *AB* or *I*, if mappings *ST* or *J* is continuous, then the proof that *z* is a common Fine Structure of *AB*, *ST*, *I* and *J* is similar.

To show that *z* is unique, Let υ be the another Fine Structure of *I*, *J*, *AB* and *ST* then

$$d(z,\upsilon) = d(ABz, ST\upsilon)$$
$$\leq \alpha_1\left[\frac{\{d(ABz,J\upsilon)\}^2 + \{d(ST\upsilon,Iz)\}^2}{d(ABz,J\upsilon)+d(ST\upsilon,Iz)}\right] + \alpha_2[d(ABz,Iz)+d(ST\upsilon,J\upsilon)]$$
$$+\alpha_3 d(Iz,J\upsilon) + F(\min\{d^2(Iz,J\upsilon), d(Iz,ABz).d(Iz,ST\upsilon),$$
$$d(J\upsilon,ST\upsilon).d(J\upsilon,ABz)\})$$
$$=(\alpha_1+\alpha_3)d(z,\upsilon) + F(\min\{d^2(z,\upsilon), d(z,z).d(z,\upsilon), d(\upsilon,\upsilon).d(\upsilon,z)\})$$

which implies that

$$d(z,\upsilon) \leq (\alpha_1+\alpha_3)d(z,\upsilon)$$

yielding there by $z = \upsilon$.

Finally we have to prove that *z* is also a common Fine Structure of *A,B,ST,I* and *J* and for this let both the pairs (*AB, I*) and (*ST, J*) having a unique common Fine Structure *z*. Then

$$Az = A(ABz) = A(BAz) = AB(Az)$$
$$Az = A(Iz) = I(Az)$$
$$Bz = B(ABz) = B(A(Bz)) = BA(Bz) = AB(Bz),$$
$$Bz = B(Iz) = I(Bz)$$

which implies that (*AB, I*) has a common Fine Structure which is *Az* and *Bz*. yielding thereby $Az=z=Bz=Iz=ABz$ in the view of uniqueness of common Fine Structure of the pair (*AB, I*).

Similarly using the commutativity of (*S,T*), (*S,J*) and (*T,J*) it in be shown that $Sz = z = Tz = Jz = STz$

Now we need to show that $Az=Sz$ ($Bz=Tz$) also remains a common Fine Structure of both the pairs (*AB,I*) and (*ST, J*) for this

$$d(Az,Sz) = d(A(BAz), S(TSz))$$
$$= d(AB(Az), ST(Sz))$$

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$$\leq \alpha_1 \left[\frac{\{d(AB(Az), J(Sz))\}^2 + \{d(ST(Sz), I(Az))\}^2}{d(AB(Az), J(Sz)) + d(ST(Sz), I(Az))} \right]$$

$$+ \alpha_2 [d(AB(Az), I(Az)) + d(ST(Sz), J(Sz))] + \alpha_3 d(I(Az), J(Sz))$$

$$+ F(\min\{d^2(I(Az), J(Sz)), d(AB(Az), I(Az)).d(ST(Sz), J(Az)),$$

$$d(AB(Az), J(Sz)).d(ST(Sz), J(Sz))\})$$

implies that $d(Az, Sz) = 0(as\ \ d(AB(Az)), J(Sz)) + d(ST(Sz), I(Az)) = 0$,
using condition (2.2)
yielding thereby $Az = Sz$. Similarly it can be shown that $Bz = Tz$.
Thus z is the unique common Fine Structure of *A,B,S,T,I,* and *J*

Case – 2:

Suppose that $d(ABx, Jy) + d(STy, Ix) = 0$ implies that $d(ABx, STy) = 0$, Then we argue as follows:
Suppose that there exists an n such that $z_n = z_{n+1}$
Then, also $z_{n+1} = z_{n+2}$, suppose not. Then from (2.3) we have
$0 < d(z_{n+1}, z_{n+2}) \leq kd(z_{n+1}, z_n)$ yielding there $z_{n+1} = z_{n+2}$. Thus $z_n = z_{n+K}$ for $k = 1,2,...$ It then follows that there exist two point w_1 and w_2 such that $\upsilon_1 = ABw_1 = Jw_2$ and $\upsilon_2 = STw_2 = Iw_1$.
Since $d(ABw_1, Jw_2) + d(STw_2, Iw_1) = 0$, from (2.3)

$d(ABw_1 STw_2) = 0 \quad i.e \quad \upsilon_1 = ABw_2 = STw_2 = \upsilon_2$

Note also that $I\upsilon_1 = I(ABw_1) = AB(Iw_1) = AB\upsilon_2$
Similarly $ST\upsilon_2 = J\upsilon_2$. Define $y_1 = AB\upsilon_1, y_2 = ST\upsilon_2$.
Since $d(AB\upsilon_1, J\upsilon_2) + d(ST\upsilon_2, J\upsilon_1) = 0$,
it follows from (2.1) that $d(AB\upsilon_1, ST\upsilon_2) = 0$ i.e. $y_1 = y_2$.
Thus $AB\upsilon_1 = I\upsilon_1 = ST\upsilon_2 = J\upsilon_2$ But $\upsilon_1 = \upsilon_2$

Therefore *AB,I,ST* and *J* have a common coincidence points define $w = AB\upsilon_1$, it then follows that w is also a common coincidence point of *AB,I, ST* and *J*, if $ABw \neq AB\upsilon_1 = ST\upsilon_1$ then d $(Abw, ST\upsilon_1) > 0$. But, since $d(ABw, J\upsilon) + d(ST\upsilon_1, Iw) = 0$. if follows from (2.2) that $d(ABw, ST\upsilon_1) = 0$, i.e. $ABw = ST\upsilon_1$, a contradiction.

Therefore $ABw = ST\upsilon_1 = w$ and w is a common Fine Structure of *AB, ST, I* and *J*.

The other part is identical to the case (1), hence it is omitted, this complete the proof.

Conclusion

Fine Structure theory is equivalent to best approximation, variation inequality and the maximal elements in mathematical economics. The sequence of iterates of Fine Structure theory can be applied to find solution of a variation inequality and the best approximation theory.

Some common Fine Structure theorems for six mappings involving rational inequalities in complete metric-spaces have been proved by using the notions of compatibility, weak compatibility and commutatively.

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A Critical Study Of Laplace Transform To Some Mathematical Problems

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Abstract

The purpose of this paper is to discuss the moment problem and non-moment problem on some applications of Laplace transforms to analytical number theory including the classical circle & divisor problem with the applications of Laplace transforms to some solutions of 2 functional equations.

Keywords: Laplace transforms, Moment problem and Non–Moment problem and Hausdroffs , Stieltjes Moment problem.

Introduction

The Laplace transform is very profitable in investigate and plan for the systems that are linear and invariant. In the beginning of 1910, these transform techniques were applied in signal processing at bell labs for the signal filtering and telephone long lines communication by H.Bode and others. After while transform theory provided the backbone of classical control theory. These were practiced during the time of world wars and up to about 1960. At that the time of 1960 the state variable techniques began to be used for control designs. A French mathematician Pierre-Simon Laplace (1749-1827) characterized by the fenchs evolution.

The name Laplace transform is derived from the French mathematician and astronomer Pierre-Simon Laplace, who used similar type of transform theory in his work on probability theory. The transform theory was widespread at the time after world war 2^{nd}. And in 19^{th} century it is used by Abel, lerch, Heaviside and Bronwich.

By the solution of differential equation we did not get the matter very far. Sir Josepd Louis Lagrange who was an admirer of Euler describes this on his work on integrating probability density functions.

In 1782 it seems that these type of integral attracted the Laplace attention in which It follows Euler spirit in using the integrals. However, In 1785 Laplace took the step forward for just looking for a solution in the form of integral. And these transform become very popular.

Akien to a Mellin transform, the whole of a differential equation just took for solution of the transformed equations. Then he started to apply Laplace transform in the same way for deriving some of its property.

The Laplace transform is one of the most widely used transform having many applications in physics and engineering. It is denoted by L{f(t)}, it is a linear operator of a function f(t) with a real argument t(t≥0) that transforms it to a function f(s) with a complex arguments. This transformation is essentially injective for the majority of practical uses. The two respective pairs of f(t) and f(s) are seen in tables Laplace transformation is profitable which mean in that many relationships and operation over the images can be correspond to simpler relationships and operations over the images. Laplace transform is derived from the value of scientist Pierre- Simon Laplace who was work on his probability theory.

The Laplace transform is also related to the Fourier transform, but the Fourier transform tells about a function or signal as a series of modes of vibrations,whereas the Laplace transform tells about a function into its moments. Like the Fourier transform, the Laplace transform is used for solve the differential equation and integral equations. In physics and engineering the Laplace transform is used for analysis of linear time invariant system such as electrical circuits, harmonic oscillator's optical devices and mechanic system. In the kind of analysis, the Laplace transform is often assumed as a transform from the time domain, in which inputs and outputs are functions of complex angular frequency. In radian per unit time. It is given that mechanical or functional description of an input or output to a system. Also the Laplace transform provides an alternative functional description that after simplify the process of analyzing the behavior of the system, or in making a new system based on a set of specifications.

MOMENT PROBLEMS

As per engineering mechanicals tools and technique, When a force of certain magnitude is applied on a particle then moment is produce, mathematically, this is measured as the product of force applied and the effective distance of the shape and size of the particle or substance taken for the pupose. Simple motto behind explaining this definition is that moment is govern by two variables and so there should be limitation of exact two variables in case of moment based problems. This fact is an eyes opener for those who consider only one variable and talk about existence of a Moment Problem only.

In case of Laplace transform generally we find that L operator is applicable on single variables as (L(sin at), L(cos at). etc what about L(sin xy), L(cosxy)?

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So for an engineering problem containing two variables is really a Non- Moment problem but Laplace transform works even a function containing a single variable, termed as is a moment problem.

So our observation is that classification of Moment is possible. So, why not a Non-Moment problem?

NON-MOMENT PROBLEMS

It is true that when a certain magnitude of force is applied on a particle, through out each and every parts of the particle there comes some sensation and a thermodynamic change is always observable. Displacement like molecular displacement should also be taken as a point of consideration.

Two variables as force and effective distance are taken in to account when defining Moment but result remains static implies that no moment is observable as looks in case of partial differentiation of the function and its results which includes exponential function. So, why only Moment problem and not non-moment problem, which highlighted and credited by Laplace as a pioneer behind invention of concept of moment problem. It is just like concept of rest and motion in the engineering mechanics, where both are relative phenomena and absolute rest is quite impossible. Let us come forward to divide engineering Mechanics problem into two parts from today only as Moment problem and Non-Moment problem.

We name a non-moment problem. Physicists may give their argument that displacement in a particle is a complete dependent factor on the magnitude of applied force. They can also support the single variable import, where Laplace seems to be agreed. This automatically indicates that there should be classification for variable bearing problem as moment problem and non moment problem. This further justifies our logic that only physical study is not enough, there is need of molecular study too. Which is locked about moment based problem.

So there should be two types of different function as according to the number of variables present in the function Sin (at), Taken by Laplace, where "a" is any arbitrary constant sin at is a moment problem (for example). for sin xy where x and y both are variables and either of one may be consideration as a constant as per the need of the solution of the problem (in case of partial differential) value of L sin(xy) is unavailable. So, sin(xy) is clearly a non-moment problem (This is only one example) most of the implicit function are the generators of non moment problem.

The method of Laplace Transform has the advantage of directly giving the solution of differential equation with boundary values without. The necessary of first finding the general solution and then evaluating from it the arbitrary constant moreover the ready tables of

Laplace Transforms reduce the problem of solving differential equation to more algebraic manipulation.

Methodology

Let $I \subseteq R$ be an interval. For a positive measure μ on *I*the nth moment is defined as $\int x^n d\mu(x)$ provided the integral exists. If we suppose that $(s_n)_{n>=0}$ is a sequence of real numbers, the moment problem on *I* consists of solving the following three problems:

(I) There exist a positive measure on *I* with moment $(s_n)_{n>=0}$.
(II) This positive measure uniquely determined by the moments$(s_n)_{n>=0}$.

When μ is a positive measure with moments $(s_n)_{n>=0}$ we say that μ is a solution to the moment problem. If the solution to the moment problem is unique, the moment problem is called determinate.otherwise the moment problem is said to be indeterminate.

(III) All positive measure on I with moments $(s_n)_{n>=0}$can we uniquely describe by the given historical reason ie. The moment problem on [0,1) is referred to as the Hausdroff's moment problem and the moment problem on R is called the Hamburger moment problem and thus $[0,\infty)$ is called the Stieltjes moment problem.

Statement of the Problem

1. **Hausdroff`s moment problem often known as moment problem is given as below-**

$$\{\mu_n\}_0^\infty : \mu_0, \mu_1, \mu_2, \ldots;$$

There are other forms of existence to predict function $\alpha(t)$ of bounded variation in interval (0,1) for example –

$$\mu_n = \int_0^1 t^n d\alpha(t) \qquad (n = 0, 1, 2, \ldots)$$

Hence, this is corollary known as moment sequence. But, it has been noticed that every sequence (3.1,1) has them form (3.1,2) since (3.1,2) implies that-

$$|\mu_n| \leqq V[\alpha(t)]_0^1$$

the quantity on the right being the variation of $\alpha(t)$ on the interval (0, 1). Thus, in this way its sequence is confined. It was F. Hausdorff [1921a] to determine essential situation of sequence to be moment sequence. But, given that representation (3.12) if $\alpha(t)$ is a normalized function of corollary differences.

$$\alpha(0)=0, \quad \alpha(t)=\frac{\alpha(t+)+\alpha(t-)}{2} \qquad (0<t<1).$$

As normalization of the function $\alpha(t)$ does not varies the value of the integral (3.1,2) so, we can take over the loss with any disturbance to general prospect $\alpha(t)$ which is normalized. Infact, the entire discussion throw light upon it without any obstacle.

Significance

In order to obtain required and significantly an adequate conditions for the representation of function as a Laplace integral we require an introductory discussion of kernels of non-negative type. These are the continuous analogues of non-negative or semi-definite quadratic forms.

According to Definition. A real function k(x,y) which is not discontinuous in the square ($\alpha < x < b$, $a < y < b$) is of non-negative type there if for every real function $\phi(x)$ continuous in ($\alpha < x < b$)

$$J(\phi) = \int_a^b \int_a^b k(x,y)\phi(x)\phi(y)\,dx\,dy \geq 0$$

For instance, take k (x, y) = g(x) g (y) where g(x) is any function continuous in ($\alpha < x < b$).

Note:

The integral (7.20,86) may finish without having $\phi(x)$ identically zero. Thus is our example we have only to choose $\phi(x)$ orthogonal to g(x) on (a,b).

A kernel is said to be non-negative definite if it is non-negative type and if integral (7.20,86) can finish for no real continuous function $\phi(x)$ except $\phi(x) = 0$. As an example take $a = 0$, $b = \pi$ and

$$k(x,y) = \sum_{n=0}^{\infty} e^{-n} \cos ns \cos ny.$$

The integral (7.20,86) becomes

$$\sum_{n=0}^{\infty} e^{-n} a_n^2$$

$$a_n = \int_0^{\pi} \phi(x)\cos nx\,dx \qquad (n = 0,1,2,......).$$

But (7.20,87) cannot be zero until all the a_n are zero. But by the completeness of the cosine set on $(0, \pi)$ this implies that $\phi(x)$ is identically zero.

To Proof : An important result of J. Mercer (1990) that brings out the connection between kernels and quadratic forms.

Axiom 7.20(a) A continuous kernel k (x,y) is of non-negative type if and only if for every non-infinite sequence $\{x_i\}_0^n$ of distinct number of $(a < x < b)$ the quadratic form-

$$Q_n = \sum_{i=0}^{n}\sum_{i=0}^{n} k(x_i, x_i)\xi_i\xi_i$$

is non-negative (definite or semi-definite).

Conclusion

This is such a partial differential equation whose solution finally contains some arbitrary constant and exponential function.

For the purpose of generating a new ideology and corollary on Non-Moment problem, we take into account equation as the standard form of an equation representing itself as the generator of non-moment problem function. To make our result more accurate and widely considerable, we would like to introduce some other facts and finding coherent to the convergence of result and making our formula as a very common discussion.

In the Engineering Mechanics, "Moment" and in the basic function theory "exponential function" have been cause of concern for those who are working on Science and Technology. Though an exponential function generate an infinite series but consequently as far as number of terms in the series increases magnitude value of such particular term recedes.

Like is the situation of impact of applied forces recedes as far as it touches molecular part of the physical substance in a Moment is produced. It is not always true to advocate that that this force plays its role in the one dimension part of the said and taken substance for the purpose of testing the Moment generated so.

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On The Harmonic Index Of $HAC_5C_6C_7$ [P,Q] Carbon Nanotube

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Abstract

Let G = (V, E) be a molecular graph with vertex set V and edge set E. The Harmonic index of a connected graph is defined as, H(G) = $\sum_{uv \in E(G)} \frac{2}{d_u + d_v}$, where d_v is the degree of a connected graph G. The Harmonic polynomial of a graph G is defined as, H(G,x) = $2\sum_{uv \in E(G)} x^{d_u + d_v - 1}$, where $\int_0^1 H(G,x)\,dx$=H(G).

In this paper we study Harmonic index, Harmonic polynomial and M-polynomial of $HAC_5C_6C_7$[p,q] carbon nanotube.

Keywords: Carbon nanotube, Harmonic index, Harmonic polynomial, M-polynomial, Molecular graph, Topological index.

1. Introduction

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. Let G(V,E) be a connected graph with vertex set V(G) and edge set E(G).The degree of a vertex u $\in V(G)$ is denoted by d_u and is the number of vertices adjacent to u. The connecting vertices u and v is denoted by uv. Topological index is a numerical value associated with chemical constitution purporting for correlation of chemical structure with various physical properties, chemical reactivity or biological activity [1].Topological indices are abundantly being used in the QSPR and QSAR researches [2].The Harmonic index is one of the most important indices in chemical and mathematical fields. Carbon nanotubes form an interesting class of carbon nonmaterial. Diudea was the first chemist who considered the problem of topological indices of nanostructures [3].

The Harmonic index gives better correlations with physical and chemical properties of molecules [4]. Degree-based topological indices of $HAC_5C_6C_7$[p,q] carbon nanotube is studied by [5].The relationships between Harmonic index and other topological indices are studied by [6-7].

The inequalities between vertex degree based topological indices is studied by [8]. Topological indices of nanotubes are studied by [9]. First and Second Zagreb polynomials of VC_5C_7[p,q] and HAC_5C_7[p,q].are studied by [10]. Zagreb polynomials of $HAC_5C_6C_7$[p,q] nanotube are studied by [11].The Harmonic polynomial and Harmonic index of certain carbon nanotubes is studied by [12].

The Harmonic polynomial and Harmonic index of a molecular graph is studied by [13]. Let G be a graph with vertex set V(G) and edge set E(G). The Harmonic index of G is defined as $H(G) = \sum_{uv \in E(G)} \frac{2}{d_u+d_v}$,where d_v is the degree of a connected graph G [14-15]. Iranmanesh et al [16] were the first to introduce the Harmonic polynomial of a Caterpillar graph G of diameter 4 as follows, $H(G,x) = 2\sum_{uv \in E(G)} x^{d_u+d_v-1}$, where $\int_0^1 H(G,x)\,dx = H(G)$.

The M-polynomial of G [17-23] is defined as,

$M(G,x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(G)\, x^i y^j$.where $\delta = \text{Min}\{d_v|v \in V(G)\}$,

$\Delta = \text{Max}\{d_v|v \in V(G)\}$,

And $m_{ij}(G)$ is the edge vu $\in E(G)$ such that $i \le j$, with $D_x = x\frac{\delta(f(x,y)}{\delta x}$, $D_y = y\frac{\delta(f(x,y)}{\delta x}$, $S_x = \int_0^x \frac{f y,t)}{t}dt$, $S_y = \int_0^y \frac{f(x,t)}{t}dt$, $Jf(x,y) = f(x,x)$, $Q_a = (f(x,y)) = x^a f(x,y)$.

The general form of family of descriptors is $D = D(G) = \sum_{u=v} F(d_u, d_v)$ and the function $F = F(x,y)$ is

$F(x,y) = \frac{2}{x+y}$ for Harmonic index formula from M-polynomial [24] see table no.1.Our notation is standard and mainly taken from standard books of graph theory [25-29].In this paper we investigate the Harmonic index, Harmonic polynomial and M-polynomial of $HAC_5C_6C_7$ [p,q] carbon nanotube.

2.Materials and Methods

A molecular graph is constructed by representing nodes and edges. The 2-dimensional lattice of $HAC_5C_6C_7$[p,q] graph is shown in figure (1).This graph consist of p rows and q periods. The number of vertices is 16pq+2p and edges are 24pq. The degree-based Harmonic index is, $H(G) = \sum_{uv \in E(G)} \frac{2}{d_u+d_v}$,where d_v is the degree of a connected graph G. Iranmanesh et al introduced the Harmonic polynomial of a caterpillar graph G of diameter 4 as follows, $H(G,x) = 2\sum_{uv \in E(G)} x^{d_u+d_v-1}$, where $\int_0^1 H(G,x)\,dx = H(G)$.

The M-polynomial of G is defined as,

$M(G,x,y) = \sum_{\delta \le i \le j \le \Delta} m_{ij}(G)\, x^i y^j$.where $\delta = \text{Min}\{d_v| v \in V(G)\}$

The Harmonic index is studied from M-polynomial by considering the function $F=F(x,y)$ as $F(x,y) = \frac{2}{x+y}$, $2S_xJ(M(G,x,y))$ see table number (1) . From molecular graph of this carbon nanotube, it is observed that there are three different edges which can be considered for studying the Harmonic index.

These are $E_{\{2,3\}}$, $E_{\{1,3\}}$ and $E_{\{2,2\}}$ as;

$E_{\{2,3\}}= \{uv\in E(HAC_5C_6C_7[p,q])|d_u=2,d_v=3\}$.

$E_{\{1,3\}} = \{uv\in E(HAC_5C_6C_7[p,q])|d_u=1,d_v=3\}$.

$E_{\{2,2\}}= \{uv\in E(HAC_5C_6C7[p,q])|d_u=2,d_v=2\}$.

The Harmonic index can be studied from degree, Harmonic polynomial and M-polynomial if the degree of each vertex and edge of a molecular graph are known.

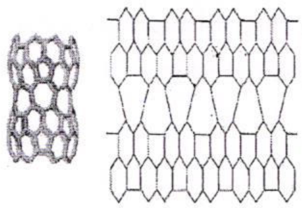

Fig.1. The 2-dimensional lattice of $HAC_5C_6C_7[p,q]$ graph

3. Results and discussion

The carbon nanotube $HAC_5C_6C_7[p,q]$ is constructed by alternating C_5,C_6 and C_7 following the trivalent decoration. The 2-dimensional lattice of $HAC_5C_6C_7[p,q]$ graph is shown in figure (1).This graph consist of p rows and q periods. Here p denotes the number of pentagons in one row and q is the number of periods. It is observed from figure that there are three separate cases where the number of edges is different [30] namely, $E_{2,3}$, $E_{1,3}$ and $E_{2,2}$. The number of vertices is 16pq+2p and edges are 24pq.

Theorem.3.1 Let $HAC_5C_6C_7$ be the graph of carbon nanotube, then its Harmonic index is equal to

$$H(HAC_5C_6C_7[p,q]) = \frac{13}{5} + \frac{1}{3}(24pq-6p).$$

Proof. The graph of the carbon nanotube $HAC_5C_6C_7[p,q]$ contains 16pq+2p vertices and 24pq edges. From fig.1 we notice that there three separate cases and the number of edges is different, namely $E_{\{2,3\}}$,$E_{\{1,3\}}$ and $E_{\{2,2\}}$.

$E_{\{2,3\}}= \{uv\in E(HAC_5C_6C_7[p,q])|d_u=2,d_v=3\}$.

$E_{\{1,3\}} = \{uv\in E(HAC_5C_6C_7[p,q])|d_u=1,d_v=3\}$.

$E_{\{2,2\}}= \{uv\in E(HAC_5C_6C_7[p,q])|d_u=2,d_v=2\}$.

The number of edges $E_{\{2,3\}}$, $E_{\{1,3\}}$ and $E_{\{2,2\}}$ are 4p,2p and 24pq-4p-2p respectively.

The $H(G) = \sum_{uv\in E(G)} \frac{2}{d_{u+}d_v}$

$= E_{\{2,3\}}\frac{2}{2+3}+E_{\{1,3\}}\frac{2}{1+3}+E_{\{2,2\}}\frac{2}{2+2}$

$=\frac{8}{5}p+p+\frac{1}{3}(24pq+-4p-2p)$

$=\frac{13}{5}p+\frac{1}{3}(24pq+-6p)$.

Theorem 3.2. Let $HAC_5C_6C_7$ be the graph of carbon nanotube, then its Harmonic polynomial and Harmonic index is equal to

$H(HAC_5C_6C_7,x)= 4px^3+2(24pq\text{-}6p)\,x^3+8px^4$,

$H(HAC_5C_6C_7)=\frac{13}{5}+\frac{1}{3}(24pq\text{-}6p)$.

Proof. The graph of the carbon nanotube $HA(C_5C_6C_7[p,q])$ contains 16pq+2p vertices and 24pq edges. From fig.1 we notice that there three separate cases and the number of edges is different, namely $E_{\{2,3\}}$, $E_{\{1,3\}}$ and $E_{\{2,2\}}$. The number of edges are $E_{\{2,3\}}=4p$, $E_{\{1,3\}}=2p$ and $E_{\{2,2\}}=24pq\text{-}4p\text{-}2p$.

Then the Harmonic polynomial,

$H(HAC_5C_6C_7,x)= 2\sum_{uv\in E(G)} x^{d_u+d_v-1}$

$= H(G,x) = 2\sum_{uv\in E\{2,3\}} x^{d_u+d_v-1} + 2\sum_{uv\in E\{1,3\}} x^{d_u+d_v-1} + 2\sum_{uv\in E\{2,2\}} x^{d_u+d_v-1}$

$=4p\ 2x^{2+3-1}+2p\ 2x^{1+3-1}+(24pq\text{-}4p\text{-}2p)\ 2x^{2+2-1}$

$=4px^3+2(24pq\text{-}6p)x^3+8px^4$.

The Harmonic index,

$HA(C_5C_6C_7)=\int_0^1 H(HAC_5C_6C_7,x)dx$

$=\int_0^1(4px^3+2(24pq-6p)x^3)+8px^4)dx$

$=px^4+\frac{1}{2}(24pq\text{-}6p)x^4+\frac{8}{5}px^5$

$=\frac{13}{5}p+\frac{1}{3}(24pq-6p)$.

Theorem 3.2. Let $HAC_5C_6C_7[p,q]$ be the graph of carbon nanotube. Then the M-polynomial $M(HAC_5C_6C_7,x,y)=4px^2y^3+2px^1y^3+(24pq\text{-}4p\text{-}2p)x^2y^2$.

Proof. Let G be the graph of $HAC_5C_6C_7[p,q]$ carbon nanotube. The edge partitions of $HAC_5C_6C_7$ are $E_{\{2,3\}}=4p$, $E_{\{1,3\}}=2p$, $E_{\{2,2\}}=24pq\text{-}4p\text{-}2p$. Thus the M-polynomial of $HAC_5C_6C_7[p,q]$ is

$M(G,x,y)=\sum_{i\le j} m_{ij}(G)x^iy^j$

$=\sum_{2\le 3} m_{23}(G)\,x^2y^3+\sum_{1\le 3} m_{13}\,(G)x^1y^3+\sum_{2\le 2} m_{22}\,x^2y^2$,

$=E_{\{2,3\}}x^2y^3+E_{\{1,3\}}x^1y^3+E_{\{2,2\}}x^2y^2$,

$=4px^2y^3+2px^1y^3+(24pq-4p-2p)x^2y^2$.

The Harmonic index is

Let $M(G,x,y)=4px^2y^3+2px^1y^3+(24pq\text{-}4p\text{-}2p)x^2y^2$.

Then $S_xJF(x,y)=\frac{4}{5}px^2y^3+\frac{2}{4}px^1y^3+\frac{1}{4}(24pq\text{-}4p\text{-}2p)x^2y^2$.

As $H(G) = 2S_x J(f(x,y))|_{x=1}$,

$M(G) = \frac{8}{5}p + p + \frac{1}{2}(24pq - 6p)$

$= \frac{13}{5}p + \frac{1}{3}(24pq - 6p)$.

Table 1. Topological index: Harmonic index from-M-polynomial.

Topological index	Derivation from M(G,x,y)
Harmonic index	$2S_xJ(M(G,x,y))$

4. Conclusion

The Harmonic index of $HA(C_5C_6C_7[p,q]$ carbon nanotube is $\frac{13}{5}p + \frac{1}{3}(24pq - 6p)$.The degree-based Harmonic index is investigated on the basis of Harmonic polynomial and M-polynomial.

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Role Of Social Engineering In Higher Education Fostering A Future Of Inclusion

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Abstract: Higher education in India has achieved remarkable progress in terms of institutions, teachers' enrolment and teaching-learning infrastructure including disciplines. The gross enrolment ratio has increased from less than 1 % in 1950 to about 10-11 % presently. It is strongly recommended that there are enormous challenges that need to be addressed immediately. The foremost priority, at this juncture appears to be enhancing access to higher education by developing a regulatory mechanism to manage the credibility of online education providers like MOOCs (massive open online courses) such that the Gross Enrolment Ratio (GER) is raised to a minimum threshold level for sustained economic development.

Key words: Higher education, MOOCs, GER, economic development

Introduction: India's Gross Enrolment Ratio (GER) in higher education stood at 21.1 % as of 2012-13 compared with the world average of 27 %. This means that we not only lag developed countries such as the US (95 %) and the UK (58 %), but also developing peers such as China (26 %), Brazil (36 %), Malaysia (40 %), Indonesia (24 %) and the Philippines (30 %). In India, according to latest All India Higher Education Survey (AIHES) reports given by HRD Ministry, the Gross Enrolment Ratio (GER) in higher education has increased from 24.5% (2015-16) to 25.2% (2016-17). A Vision Plan of 5 years titled as Education Quality Upgradation and Inclusion Programme (EQUIP) is finalized and released by the Higher Education Department of the Union Ministry of Human Resource Development. EQUIP report with 10 focus areas has been prepared after a detailed exercise done by Experts covering following ten areas:

1. Strategies for expanding access
2. Towards global best teaching/learning process
3. Promoting Excellence
4. Governance reforms
5. Assessment, Accreditation & Ranking systems
6. Promotion of research & innovation
7. Employability & entrepreneurship
8. Using Technology for better reach
9. Internationalisation
10. Financing higher education

For above mentioned 10 focus areas, 10 Expert Group (one for each) drawn from senior academicians, administrators and industrialists suggested over 50 initiatives that would transform country's higher education sector completely. Following goals are set for Higher Education Sector by Expert Groups:

1. Doubling Gross Enrolment Ratio (GER) and resolving geographically and socially skewed access to higher education institutions in India.
2. Promoting India as a global study destination.
3. Upgrade Quality of Education to global standards.
4. Positioning minimum 50 Indian institutions among Top-1000 Global Universities.
5. Promote Research & Innovation ecosystems for positioning India in Top-3 countries globally in matters of knowledge creation
6. Introducing governance reforms in higher education for well-administered campuses.
7. For assurance of quality, accreditation of all institutions.
8. Double the employability for students qualifying in higher education.
9. Harness education technology for growing reach and improving teaching method.
10. Achieving a enormous increase in investments in higher education.

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Way Forward

For the above mentioned initiatives, the Expert Groups recommended modalities for investments, implementation and timelines. Through EFC (Expenditure Finance Committee) mechanism this proposal may now be forwarded to inter-departmental consultations and appraisal before taking it to Cabinet for approval.

Only expansion of infrastructure of institutions and intake capacity shall not necessarily make higher education inclusive. Expansion in higher education should be made in such a manner that it offers equitable access to all. There is need to have conscious effort to ensure that the higher educational avenues and opportunities are made available to all and that the system does not suffer on account of disparities across region, gender, social groups.(such as scheduled castes, scheduled tribes, other backward castes, minorities, physically challenged and poor). India would soon have the largest youth population in the world but it would be unskilled. Thus there is need to develop more effective lifelong learning. GOI started initiatives for skill development and has a number of skill training programmes but these still need to scale up. GOI alone could not have adequate resources for scaling it up. It needs to look at a fundamental restructuring of educational delivery system and opportunity for skill development. Delores Report (1996) a holistic education includes

- Learning to live together;
- Learning to know;
- Learning to do;
- Learning to be; and
- Learning to transform oneself and society

Educational framework should facilitate link between education for living and education for making a living. This vision strengthens the new notion of citizenship where notions of skill development, capacity enhancement and training are reformulated in terms of rights-based approach. Thus education should reach beyond theory. A shift towards new integrated education system is needed to introduce key competencies which are essential for successfully shaping one's own life and creating a functional society.

Understanding Inclusive Approach

In developing countries particularly in south East Asia, social and economic exclusion and inclusion have recently become the focus of attention among those who are concerned about equality and justice, and also for the inclusive growth. This is an approach to improve our vision. It also provides a new way to look at the root causes of old problems, like discrimination, disadvantage, and disability. It should be noted that inclusive approach is a better tool for analyzing policies, programs, legislation and practices to determine whether they promote the economic and social inclusion of individuals, families, and communities. It opens door to innovative thinking and opens up minds to new solutions for old problems. Ultimately, it provides a new way to encourage change that will transform society towards equality and justice. The Inclusive approach helps stakeholders (policy makers, program managers, and community leaders) who work in the context of social and economic exclusion. It provides a method for analyzing both the conditions of exclusion and solutions that promote inclusion. Moreover, it provides an approach of beginning to plan for inclusion.

Pathways and Inclusive Approach

A pathway is a programme of capacity/skill building for the excluded group of students and is based on the philosophy of inclusion. Masterminds of pathways have evolved programme to create such innovative activities that are inclusive in the sphere of education and career and hence towards capacity building of excluded. These interventions help them translate the concepts of social and economic exclusion and inclusion into concrete terms that can then be fed into the future policy development process. This programme has provided a way to begin the dialogue with excluded groups, raise awareness about how exclusion works, and identify steps to move toward policies, programs, and practices that will be inclusive.

Dimensions of Exclusion and Inclusion

There are enormous elements to inclusion and exclusion that should be considered in analyzing a policy, program, or practice. It should be noted that economic and social inclusion and exclusion can be observed along several dimensions like economic, functional, cultural, participatory, political, structural, physical and relational. It is not intended to be a complete list, but to stimulate to think about which of these may apply to a particular situations. Few of the elements may relate to more than one dimension.

Guiding Principles of Pathways

- Focus on the Underprivileged
- Holistic Development
- Linking individual with the society
- Implementation in partnership
- Building assets for society
- Inclusive Growth

Soft Skills and Value Added Programme (SVAP): Highly talented students, particularly from disadvantaged group, used to miss out many opportunities because their English language proficiency does not match the expected standard. This challenge has been met with most successfully by Pathways. The exceptional result of the students, year after year, bears testimony to this fact. We are constantly looking for new ways to help students to develop their communication skill.

Soft Skills -a key to better Career

Reports of the well documented studies conducted by Harvard University and Stanford Research Institute shows that technical skills and knowledge contribute to a limit of 15 per cent of one's success whereas soft skills make up the remaining 85 per cent. Soft skills include Self Development Skills, Interaction Skills Organization Skills and Communication Skills. Soft Skills and Personal Qualities (Innate Skills) with personal values shows a person's ability to fit into a given situation. Soft skills are beneficial for creating and taking advantage of opportunities, it may be a fresh job or making a long-term career or becoming a professional or businessman. Interestingly, achieving soft skills also empowers anyone to build flexibility into future. In today's scenario, in order to make a successful career and to lead a quality life soft skills have proved to be prerequisite. Demand driven, simple well designed and structured syllabi of SVAP includes basic dimensions of personal enhancement like Emotional Intelligence, Transactional Styles, Motivation, Time Management, interview skills, team work etc. It is designed with intense consultations with all stakeholders, primarily to cater the perceived needs of "BACKWARD" students in order to ensure their access to excellence. Curriculum is beyond theory and transforms their mental attitudes, habits and frame of thought and makes them ready for industry. This proved as a facilitating link between education for living and education for making living. It helps them to respect diversity and cultural heritage sensitizing towards social concerns like environment, gender, human rights, RTI etc. This is an ever-evolving phenomenon for lifelong learning and SVAP under Pathways keep abreast with the latest methodology thus ensuring that the best in each student can be bring out. SVAP are felt as Graduation++ , Ensuring sustainable source of supplementary education in many of the partners as regular up to six month long term module. It has gained popularity simply by word of mouth. Off-campus learners also shows interest to join. Enabling the so called developmental Laggards to strengthen themselves to achieve excellence in their performance and succeed in their chosen field of work. At least it raises their confidence level with exposure to these ideas. It provides a unique opportunity for the students on to how to develop their personality and upgrade their soft skills thus enhancing their career prospects. Participants get abundant scope to interact with each other and experience a wide variety of issues, topics, and situations that they are likely to encounter in future.

Conclusion

The success of the experience inspires one to share the experience and have introspection for developing better strategies and vision for future plan of action. Affirmative action should always be preferred with the inclusion of all the dimensions of personal skill building. Partnership and networking is always better option to provide excellent inputs for SVAP. SVAP should always be paid programme even if you can afford to provide it free. Update technologies and interactive teaching methodologies are more effective in transferring skill. National events provide wide exposure to learn and get encouragement for upward movement.

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Initial Value Problem of Fractional Differential Equation

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Abstract:

This paper is mainly concerned with the existence the solution for an initial value problem of nonlinear fractional differential inclusion by using a fixed point theorem. Herein, we discuss the existence result for fractional differential equation.

Keywords: Banach algebras, hybrid fixed point theorem, functional differential equation, existence result, locally attractive solution, Extremal solution.

Introduction

Fractional Calculus which deals with the derivatives and integrals of arbitrary orders, is gaining much interest because the topic has found numerous miscellaneous applications connected with real world problems as they appear in many fields of physics, mechanics, chemistry, and engineering [1–3]. Another factor attracting the attention of many scientists is the nonlocal nature of fractional-order operators which accounts for the hereditary properties of many materials and processes.

On the other hand realistic problems arising in economics, optimal control and so on can be modeled by differential inclusions and so differential inclusions are widely investigated by many authors.

Given a closed and bounded interval $J=[0,1]$ of the R_+. Consider initial value nonlinear differential inclusion of fractional order [IVNDI]given by,

$$\left.\begin{array}{c} D^{\xi}\left[\dfrac{x(t)}{f(t,x(t),x(\gamma(t)))}\right]\in F(t,x(t),x(\mu(t))) \\ x(0)=0,\ \ \xi\in(0,1) \end{array}\right\} \quad (1.1)$$

Where, D^{ξ} is Riemann-Liouville derivative, $:\mathbb{J}\times\mathbb{R}\times\mathbb{R}\to\mathbb{R}-\{0\}$, $F:\mathbb{J}\times\mathbb{R}\times\mathbb{R}\to \mathrm{P}\ (\mathbb{R})$,is a multivalued map and $\mathrm{P}\ (\mathrm{R})$ is the set of all non-empty sub-sets of$\mathbb{R}$, and $\gamma,\mu:\mathbb{R}_+\to\mathbb{R}$

The main objective of the present study is to establish an existence result for the problem (1.1) underLipschitz and Caratheodory conditions by applying a fixed point theorem in Banach algebras.

1. Preliminaries:

Fractional Calculus :

Definition 2.1[1]: Let$f\in L^1[0,T]$and>0. The Riemann – Liouville fractional derivative of order ξ of real function f is defined as $D^{\xi}f(t)=\frac{1}{\Gamma(1-\xi)}\frac{d}{dt}\int_0^t\frac{f(s)}{(t-s)^{\xi}}ds$, $0<\xi<1$Such that $D^{-\xi}f(t)=I^{\xi}f(t)=\frac{1}{\Gamma(\xi)}\int_0^t\frac{f(s)}{(t-s)^{1-\xi}}ds$respectively.

Definition 2.2[1]: The Riemann-Liouville fractional integral of order $\xi\in(0,1)$ of the function $f\in L^1[0,T]$is defined by the formula: $I^{\xi}f(t)=\frac{1}{\Gamma(\xi)}\int_0^t\frac{f(s)}{(t-s)^{1-\xi}}ds,\ \ t\in[0,\mathbb{T}]$

whereΓdenote the Euler gamma function.

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The Riemann-Liouville fractional derivative operator of order ξ defined by $D^{\xi} = \frac{d^{\xi}}{dt^{\xi}} = \frac{d}{dt} \circ I^{1-\xi}$
It may be shown that the fractional integral operator I^{ξ} transforms the space $L^1(\mathbb{R}_+, \mathbb{R})$ into itself and has some other properties

Multivalued Analysis:

Let us recall some basic definitions on multivalued maps [15, 16],

For a normed linear space $(X, \|.\|)$, Let $P_{cl}(X) = \{Y \in P(X) : Y \text{ is closed}\}$ $P_b(X) = \{Y \in P(X) : Y \text{ is bounded}\}$, $P_{cp}(X) = \{Y \in P(X) : Y \text{ is compact}\}$, $P_{cp,cv}(X) = \{Y \in P(X) : Y \text{ is compact and convex}\}$

Definition 2.3 [15, 16]: A multivalued map $G : X \to P(X)$ is convex (closed) valued if $G(x)$ is convex(closed)for all $x \in X$.

Definition 2.4 [15, 16]: The map G is bounded on bounded sets if $G(B) = \bigcup_{x \in B} G(x)$ is bounded in X for all $B \in P_b(X)$ (i.e., $\sup_{x \in B}\{\sup\{|y| : y \in G(x)\}\} < \infty$).

Definition 2.5[15, 16]: Let $\mathbb{X}$ normed linear space with norm $|.|$, $P(X)$ is the set of all non-empty sub-sets of $\mathbb{X}$ and multivalued map $G : X \to P(X)$. Then

1. G is said to be upper semi continuous (u.s.c.) on $\mathbb{X}$ if for each $x_0 \in X$, the set $G(x_0)$ is a nonempty closed subset of X, and for each open set N of X containing $G(x_0)$, there exists an open neighbourhood N_0 of x_0 such that $G(N_0) \subseteq N$
2. G is said to be completely continuous if $G(B)$ is relatively compact for every $B \in P_b(X)$
3. Gis completely continuous with nonempty compact values, then Gis u.s.c. if and only if $x_n \to x_*$, $y_n \to y_*$, and $y_n \in G(x_n)$ imply $y_* \in G(x_*)$.
4. G has a fixed point if there is $x \in X$ such that $x \in G(x)$. The fixed point set of the multivalued operator G will be denoted by Fix G.

Definition 2.6[15, 16]: A multivalued map $G : [0;1] \to P_{cl}(\square)$ is said to be measurable if for every $y \in \square$ the function $t \mapsto d(y, G(t)) = \inf\{|y - z|; z \in G(t)\}$ is measurable.

Let $C(R_+, R)$ denote a Banach space of continuous functions from R_+ to $\square$ with the norm $\|x\| = \sup_{t \in R_+} |x(t)|$. Let $L^1(R_+, R)$ be the Banach space of measurable functions $x : R_+ \times R \times R \to R$ which are Lebesgue integrable and normed by $\|x\|_{L^1} = \int_0^1 |x(t)| dt$

Definition 2.7: A multivalued map $F : \square_+ \times \square \times \square \to P(\square)$ is said to be Caratheodory if

(i) $t \mapsto F(t, x(t), x(\mu(t)))$ is measurable for each $x \in R$;

(ii) $t \mapsto F\left(t, x(t), x(\mu(t))\right)$ is upper semicontinuous for almost all $t \in J$

Further, a Caratheodory function F is called L^1Caratheodory if

(iii) There exists a function $v \in L^1(\mathrm{R}_+, \mathrm{R})$ such that $\left\|F\left(t, x(t), x(\mu(t))\right)\right\| = \sup\left\{|g|; g \in F\left(t, x(t), x(\mu(t))\right)\right\} \le v(t)$ for all $x \in \mathrm{R}$ and for a. e. $t \in J \times \mathrm{R} \times \mathrm{R}$

For each $y \in C\left([\mathrm{R}_+ \times \mathrm{R} \times \mathrm{R}], \mathrm{R}\right)$ define the set of selections of F by $S_{F,y} = \left\{g \in L^1(J, \mathrm{R}) : g(t) \in F\left(t, y(t), y(\gamma(t))\right) \, for \; a.\, e.t \in \mathrm{R}_+\right\}$

The following Lemma is used in the sequel.

Lemma 2.1(see[17]): Let X be a Banach space. Let $F : \mathrm{R}_+ \times \mathrm{R} \times \mathrm{R} \to \mathrm{P}_{cp,cv}(X)$ be an L^1 Caratheodory multivalued map and let Θ be a linear continuous mapping from $L^1\left([\mathrm{R}_+ \times \mathrm{R} \times \mathrm{R}], X\right)$ to $C\left([\mathrm{R}_+ \times \mathrm{R} \times \mathrm{R}], X\right)$, $x \mapsto (\Theta \circ S_F)(x) = \Theta(S_{F,x})$ is a closed graph operator in $C\left([\mathrm{R}_+ \times \mathrm{R} \times \mathrm{R}], X\right) \times C\left([\mathrm{R}_+ \times \mathrm{R} \times \mathrm{R}], X\right)$.

The following fixed point theorem due to fundamental in the proof of our main result.

Lemma 2.2: Let X be a Banach algebra, let $A : X \to X$ be a single valued, and let $B : X \to \mathrm{P}_{cp,cv}(X)$ be a multivalued operator satisfying the following:

a) A is single valued Lipschitz with a Lipschitz constant k ,
b) B is compact and upper semi continuous,
c) $2Mk < 1$ where $M = \|B(X)\|$.

Then either

(i) The operator inclusion $x \in AxBx$ has a solution or
(ii) The set $\varepsilon = \{u \in X \mid \mu u \in AuBu, \mu > 1\}$ is unbounded.

2. Main Result :

Definition 3.1:A function $x \in AC^1(\mathrm{R}_+ \times \mathrm{R} \times \mathrm{R}, \mathrm{R})$ is called a solution of the problem (1.1) if $\exists$ a function $g \in L^1(\mathrm{R}_+ \times \mathrm{R} \times \mathrm{R}, \mathrm{R})$ with $g(t) \in F\left(t, x(t), x(\mu(t))\right)$ a. e. on R_+ such that

$$\left.\begin{aligned} & D^{\xi}\left[\frac{x(t)}{f\left(t, x(t), x(\gamma(t))\right)}\right] = g\left(t, x(t), x(\mu(t))\right), t \in J \\ & x(0) = 0, \xi \in (0,1) \end{aligned}\right\} \tag{3.1}$$

Lemma 2.4.3: Suppose that $\xi \in (0,1)$ and the function f, g satisfying IVNDI (1.1) then x is the solution of the IVNDI (1.1) if and only if it is the solution of integral inclusion

$$\left.\begin{aligned} x(t) \in \left[f\left(t, x(t), x(\gamma(t))\right)\right]\left[\frac{1}{\Gamma(\xi)}\int_0^t (t-s)^{\xi-1} F\left(s, x(s), x(\mu(s))\right) ds\right], t \in J \\ x(0) = 0, \xi \in (0,1) \end{aligned}\right\} \tag{3.2}$$

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Proof: Integrating equation (3.1) of fractional order ξ, we get,

$$D^{\xi} I^{\xi}\left[\frac{x(t)}{f\left(t, x(t), x(\gamma(t))\right)}\right] \in I^{\xi}\left[F\left(s, x(s), x\left(\mu(s)\right)\right)\right]$$

$$x(t) \in \left[f\left(t, x(t), x(\gamma(t))\right)\right]\left[I^{\alpha} F\left(s, x(s), x\left(\mu(s)\right)\right)\right]$$

$$\in \left[f\left(t, x(t), x(\gamma(t))\right)\right] \times \left[\frac{1}{\Gamma(\xi)} \int_0^t (t-s)^{\xi-1} F\left(s, x(s), x\left(\mu(s)\right)\right) ds\right],$$

$$x(0) = 0, t \in J$$

Since $\int_0^t f(t) dt^n = \int_0^t \frac{(t-s)^{n-1}}{(n-1)!} f(s) ds$ Where $n = 0,1,2,3, \dots \dots \dots$

Conversely differentiate (3.2) of order ξ w.r.to t, we get,

$$D^{\xi}\left[\frac{x(t)}{f\left(t, x(t), x(\gamma(t))\right)}\right] \in D^{\xi}\left\{\frac{1}{\Gamma(\xi)} \int_0^t (t-s)^{\xi-1} F\left(s, x(s), x\left(\mu(s)\right)\right) ds\right\}$$

$$\in D^{\xi} I^{\xi} F\left(t, x(t), x\left(\mu(t)\right)\right)$$

$$\in F\left(t, x(t), x\left(\mu(t)\right)\right)$$

We consider the following set of hypotheses in the sequel.

(H_1) The function $f: J \times \mathrm{R} \times \mathrm{R} \to \mathrm{R} - \{0\}$ is continuous and there exist a bounded function $k: \mathrm{R}_+ \to \mathrm{R}$ with bound K satisfying

$$\left|f\left(t, x(t), x(\gamma(t))\right) - f\left(t, y(t), y(\gamma(t))\right)\right| \le k(t) \max\left\{\left|x(t) - y(t)\right|, \left|x(\gamma(t)) - y(\gamma(t))\right|\right\}$$

a.e. $t \in J$ $\forall x, y \in \square$

(H_2) The function $F: J \times \square \times \square \to \mathrm{P}(\square)$ is L^1 Caratheodory and has non empty compact and convex values

(H_3) There exists a positive real number R such that $R > \dfrac{F_0 / \Gamma(\xi) \|v\|_{L^1}}{1 - \left(\|K\| / \Gamma(\xi)\right)\|v\|_{L^1}}$ where

$\left(\|K\| / \Gamma(\xi)\right)\|v\|_{L^1} < \frac{1}{2}$, $F_0 = \sup_{t \in \square_+ \times \square \times \square} F(t, 0, 0)$

Theorem 3.1: Suppose that the hypotheses(H_1)- (H_3) are hold. Then the [IVNDI] (1.1) has a solution on J.

Proof: Let set $X = C\left(J \times \square \times \square, \square\right)$ be a Banach Algebras. By a solution of IVNDI (1.1) we mean a continuous function $x: J \to \mathbb{R}$ that satisfies IVNDI (1.1) on J.

Now the IVNDE(1.1) is equivalent to the IVNII (3.2)

Now we define two operators $A: X \to X$ by $Ax(t) = f\left(t, x(t), x\left(\gamma(t)\right)\right)$ $t \in J$ and

$$Bx(t) = \left\{h \in C(J \times \square \times \square . \square) : h = \left(\frac{1}{\Gamma(\xi)} \int_0^t (t-s)^{\xi-1} . g\left(s, x(s), x\left(\mu(s)\right)\right) ds\right)\right\}$$

where $S_{F,x} = \left\{ g \in L^1(J,\mathrm{R}) : g(t) \in F(t, x(t), x(\mu(t))) \; for \; a.\,e.t \in J \right\}$

Then the IVNII (3.2) is equivalent to the operator inclusion $x \in A(x)B(x)$.

We shall show that the operators A and B satisfy all the conditions of Lemma(2.2) on J.

Step I: A is Lipschitz on X that is (a) of Lemma (2.2) holds

Let $x, y \in X$ by H_1 we have $\left| Ax(t) - Ay(t) \right| = \left| f(t, x(t), x(\gamma(t))) - f(t, y(t), y(\gamma(t))) \right|$

$\leq k(t) \max\left\{ \left| x(t) - y(t) \right|, \left| x(\gamma(t)) - y(\gamma(t)) \right| \right\}$

Taking supremum over t we get

$\|Ax - Ay\| \leq \|K\| \|x - y\| \;\forall x, y \in X$

Thus A is Lipschitz with Lipschitz constant K

Step II: The multivalued operator B is compact and upper semi-continuous on X that is (b) of Lemma (2.2) holds

First we will show that B has convex values ,Let $u_1, u_2 \in Bx$ then there are $g_1, g_2 \in S_{F,x}$ such that

$$u_1(t) = \frac{1}{\Gamma(\xi)} \int_0^t (t-s)^{\xi-1} g_1(s, x(s), x(\mu(s))) ds$$

$$u_2(t) = \frac{1}{\Gamma(\xi)} \int_0^t (t-s)^{\xi-1} g_2(s, x(s), x(\mu(s))) ds$$

For any $\theta \in [0,1]$ we have $\theta u_1 + (1-\theta) u_2 = \frac{1}{\Gamma(\xi)} \int_0^t (t-s)^{\xi-1} \theta g_1 + (1-\theta) g_2 ds$

$$= \frac{1}{\Gamma(\xi)} \int_0^t (t-s)^{\xi-1} \bar{g} ds$$

Where $\bar{g} = \theta g_1 + (1-\theta) g_2$, $\bar{g} \in F(t, x(t), x(\gamma(t)))$ for all $t \in J$

Hence $\theta u_1 + (1-\theta) u_2 \in Bx$ and consequently Bx is convex for each $x \in X$

As a result B defines a multivalued operator $B : X \to \mathrm{P}_{cv}(X)$

Next we will show that B maps bounded sets into bounded sets in X.

To see this let Q be a bounded set in X . Then $\exists$ a real number $r > 0$ such that $\|x\| < r \;\forall x \in Q$.

Now for each $h \in Bx$ $\exists$ a $g \in S_{F,x}$ such that $h(t) = \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g(s, x(s), x(\mu(s))) ds$

Then for each $t \in J$ using (H_2) we have $\left| Bx(t) \right| = \left| \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g(s, x(s), x(\mu(s))) ds \right|$

by (H_2) i.e. F is L^1 Caratheodory there exists a v such that

$$\left| Bx(t) \right| \leq \left| \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} v(s, x(s), x(\mu(s))) ds \right|$$

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$$\leq \frac{1}{\Gamma(\xi)}\|v\|_{L^1}$$

This further implies that $\|h\| \leq \frac{1}{\Gamma(\xi)}\|v\|_{L^1}$ and so $B(x)$ is uniformly bounded

Next we will show that B maps bounded sets to equicontinuous sets.

Let Q be as above a bounded set and $h \in Bx$ for some $x \in Q$ then $\exists$ a $g \in S_{F,x}$ such that

$$h(t) = \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds$$

Then for any τ_1 and $\tau_2 \in \mathbb{R}_+$ we have

$$|h(\tau_2) - h(\tau_1)| = \left| \frac{1}{\Gamma(\xi)} \times \int_0^{\tau_2} (\tau_2 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds - \frac{1}{\Gamma(\xi)} \times \int_0^{\tau_1} (\tau_1 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds \right|$$

$$\leq \frac{1}{\Gamma(\xi)} \left| \int_0^{\tau_2} (\tau_2 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds - \int_0^{\tau_1} (\tau_1 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds \right|$$

$$\leq \frac{1}{\Gamma(\xi)} \left| \begin{array}{l} \int_0^{\tau_2} (\tau_2 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds - \int_0^{\tau_2} (\tau_1 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds \\ + \int_0^{\tau_2} (\tau_1 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds - \int_0^{\tau_1} (\tau_1 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds \end{array} \right|$$

$$\leq \frac{1}{\Gamma(\xi)} \left| \int_0^{\tau_2} (\tau_2 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds - \int_0^{\tau_2} (\tau_1 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds \right|$$

$$+ \frac{1}{\Gamma(\xi)} \left| \int_0^{\tau_2} (\tau_1 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds - \int_0^{\tau_1} (\tau_1 - s)^{\xi-1} g(s, x(s), x(\mu(s)))\,ds \right|$$

$$\leq \frac{1}{\Gamma(\xi)} \left| \int_0^{\tau_2} (\tau_2 - s)^{\xi-1} v(s)\,ds - \int_0^{\tau_2} (\tau_1 - s)^{\xi-1} v(s)\,ds \right|$$

$$+ \frac{1}{\Gamma(\xi)} \left| \int_0^{\tau_2} (\tau_1 - s)^{\xi-1} v(s)\,ds - \int_0^{\tau_1} (\tau_1 - s)^{\xi-1} v(s)\,ds \right|$$

$$\leq \frac{\|v\|_{L^1}}{\Gamma(\xi)} \left\{ \left| \int_0^{\tau_2} (\tau_2 - s)^{\xi-1} - (\tau_1 - s)^{\xi-1}\,ds \right| + \left| \int_{\tau_1}^{\tau_2} (\tau_1 - s)^{\xi-1}\,ds \right| \right\}$$

$$\leq \frac{\|v\|_{L^1}}{\Gamma(\xi)} \left\{ \left[\frac{(\tau_2 - s)^{\xi}}{\xi} \right]_0^{\tau_2} - \left[\frac{(\tau_1 - s)^{\xi}}{\xi} \right]_0^{\tau_2} + \left[\frac{(\tau_1 - s)^{\xi}}{\xi} \right]_{\tau_1}^{\tau_2} \right\}$$

$$\leq \frac{\|v\|_{L^1}}{\xi \times \Gamma(\xi)} \left\{ \left[(\tau_2 - \tau_2)^{\xi} - (\tau_2 - 0)^{\xi} \right] - \left[(\tau_1 - \tau_2)^{\xi} - (\tau_1 - 0)^{\xi} \right] + \left[(\tau_1 - \tau_2)^{\xi} - (\tau_1 - \tau_1)^{\xi} \right] \right\}$$

$$\leq \frac{\|v\|_{L^1}}{\Gamma(\xi+1)} \left\{ \left[-(\tau_2)^{\xi} \right] - \left[(\tau_1 - \tau_2)^{\xi} - (\tau_1)^{\xi} \right] + \left[(\tau_1 - \tau_2)^{\xi} \right] \right\}$$

$$\leq \frac{\|v\|_{L^1}}{\Gamma(\xi+1)} \left\{ (\tau_1)^{\xi} - (\tau_2)^{\xi} \right\}$$

The right hand side of the above inequality does not depend on X and tends to zero

Therefore $\left| Bx(\tau_2) - Bx(\tau_1) \right| \to 0$ as $\tau_1 \to \tau_2$

Therefore it follows by the Arzela-Ascoli theorem that $B : X \to \mathrm{P}(X)$ is completely continuous.

In our next step we will show that B has a closed graph.

Let $x_n \to x$, $h_n \in B(x_n)$ and $h_n \to h_*$ then we need to show that $h_* \in B$

Associated with $h_n \in B(x_n)$ there exists a $g_n \in S_{F,x_n}$ such that for each $t \in \square_+$

$$h_n(t) = \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g_n\left(s, x(s), x(\mu(s))\right) ds$$

Thus it suffices to show that $\exists\ g_* \in S_{F,x_*}$ such that

$$h_*(t) = \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g_*\left(s, x(s), x(\mu(s))\right) ds$$

Let us consider the linear operator $\Theta : L^1(\square_+ \times \square \times \square, \square) \to C(\square_+ \times \square \times \square, \square)$ given by

$$f \mapsto \Theta g(t) = \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g\left(s, x(s), x(\mu(s))\right) ds$$

Observe that $\left\| h_n(t) - h_*(t) \right\| = \left\| \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} (g_n - g_*) ds \right\| \to 0$ as $n \to \infty$

Thus it follows by Lemma (2.1) that $\Theta \circ S_{F}$ is a closed graph operator

Further we have $h_n(t) \in \Theta\left(S_{F,x_n}\right)$ since $x_n \to x_*$ therefore we have

$$h_*(t) = \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g_*\left(s, x(s), x(\mu(s))\right) ds \quad \text{For some } g_* \in S_{F,x_*}$$

As a result we have that the operator B is compact and upper semi-continuous operator on X .

Step III: Now we will show that $2Mk < 1$ that is (c) of Lemma (2.2) holds

This is obvious by (H_3) since we have

$M = \left\| B(X) \right\| = \sup\left\{ |Bx| ; x \in X \right\} \leq {}^{1}\!/_{\Gamma(\xi)} \|v\|_{L_1}$ and K is bound by (H_2)

Thus all the conditions of Lemma (2.2) are satisfied and a direct application of it yields that either conclusion (i) or conclusion (ii) holds. We will show that conclusion (ii) is not possible

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Let $u \in E$ be arbitrary. Then we have for $\lambda > 1$ $\lambda u \in Au(t)\, Bu(t)$

Then $\exists$ $g \in S_{F,x}$ such that for any $\lambda > 1$ one has

$$u(t) = \lambda^{-1} f\left(t, u(t), u(\gamma(t))\right) \times \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g\left(s, u(s), u(\mu(s))\right) ds \text{ for all } t \in J$$

Then we have

$$|u(t)| = \lambda^{-1} \left| f\left(t, u(t), u(\gamma(t))\right) \right| \times \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g\left(s, u(s), u(\mu(s))\right) ds$$

$$\leq \left[\left| f\left(t, u(t), u(\gamma(t))\right) - f(t,0,0) + f(t,0,0) \right| \right] \times \frac{1}{\Gamma(\xi)} \times \int_0^t (t-s)^{\xi-1} g\left(s, u(s), u(\mu(s))\right) ds$$

$$\|u\| \leq \left(K\|u\| + F_0 \right) \frac{1}{\Gamma(\xi)} \|v\|_{L^1}$$

Where we have put $F_0 = \sup_{t \in J \times \mathbb{R} \times \mathbb{R}} |f(t,0,0)|$ then with $\|u\| = R$, we have

$$R \leq \frac{F_0 / \Gamma(\xi) \|v\|_{L^1}}{1 - \left(K / \Gamma(\xi) \right) \|v\|_{L^1}}$$

Thus the condition (ii) of Lemma (2.2)does not hold .

Therefore the operator inclusion $x \in AxBx$ and consequently problem (1.1) have a solution on J

This completes the proof.

Example:Consider the initial value problem

$$D^{1/2} \left[\frac{x(t)}{\frac{\sin 2t}{3} \left[\frac{x(t)}{1-x(t)} + \log t \right] + 1} \right] \in F\left(t, x(t), x(\mu(t))\right) 0 < t < 1 \xi = \frac{1}{2} \quad \gamma = t \quad \mu = t$$

Where $F: \mathbb{J} \times \mathbb{R} \times \mathbb{R} \to P(\mathbb{R})$ is a multivalued map given by

$$t \to F\left(t, x(t), x(\mu(t))\right) = \left[\frac{|x|^3}{10(|x|^3+1)}, \frac{|\sin x|}{9(|\sin x|+1)} + \frac{2}{7}, \frac{|\sin 3x|}{|\sin 3x| + 5} \right]$$

Now,

(H1) $\left| f\left(t, x(t), x(\gamma(t))\right) - f\left(t, y(t), y(\gamma(t))\right) \right|$

$$= \left| \left\{ \frac{\sin 2t}{3} \left[\frac{x(t)}{1-x(t)} + \log t \right] + 1 \right\} - \left\{ \frac{\sin 2t}{3} \left[\frac{y(t)}{1-y(t)} + \log t \right] + 1 \right\} \right|$$

$$= \left| \frac{\sin 2t}{3} \left[\frac{x(t)}{1-x(t)} - \frac{y(t)}{1-y(t)} \right] \right|$$

$$\leq \left| \frac{\sin 2t}{3} \right| \left| \frac{x(t)y(t) + x(t) - y(t) - x(t)y(t)}{x(t)y(t) - x(t) - y(t) + 1} \right|$$

$$\leq \left| \frac{\sin 2t}{3} \right| |x(t) - y(t)|$$

$$\leq k(t)|x(t) - y(t)|$$

$$\leq K|x(t) - y(t)|$$

Since $k(t) = \frac{sin2t}{3}$ say which has bound Kon $\mathbb{R}_+$.

(H₂) and (H₃) $\left\| F\left(t, x(t), x(\mu(t))\right) \right\| = sup\left\{|g|; g \in F\left(t, x(t), x(\mu(t))\right)\right\} \leq v(t)$

$$f \in F\left(t, x(t), x(\gamma(t))\right)$$

$$|f| \leq max\left(\frac{|x|^3}{10(|x|^3+1)}, \frac{|sinx|}{9(|sinx|+1)} + \frac{2}{7}, \frac{|sin3x|}{|sin3x|+5}\right) \leq 1 \quad x \in \mathbb{R}$$

$$\left\| F\left(t, x(t), x(\mu(t))\right) \right\| = sup\left\{|y|: y \in F\left(t, x(t), x(\mu(t))\right)\right\} \leq 1 = v(t) \quad x \in \mathbb{R}$$

Clearly$\frac{\|K\|}{\Gamma(1/2)} \|v\|_{L^1} < \frac{1}{2}$

And $R > \frac{F_0 / \Gamma(\xi) \|v\|_{L^1}}{1 - (\|K\| / \Gamma(\xi)) \|v\|_{L^1}}$

$$R > \frac{3}{10(3\sqrt{\pi} - 1)}$$

Hence all the conditions of Theorem are satisfied, and, accordingly, the above problem has a solution on.

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A Significant Study of Laplace Integral Transform

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Abstract:

This present paper deals with Laplace integral transform, Laplace inverse integral formula, define analytic function, Region of convergence, Integration by parts, determining function of Laplace integrals.

Keywords: Laplace integral transform, Inversionformula, Analyticfunction, Normalizedfunctions, Convergence, Bilaterallaplaceintegral, Laurent series

Introduction

The Laplace transform is one of the most widely used transform having many applications in physics and engineering. It is denoted by $L\{f(t)\}$, it is a linear operator of a function f(t) with a real argument $t(t \geq 0)$ that transforms it to a function f(s) with a complex arguments. This transformation is essentially bifective for the majority of practical uses. The two respective pairs of f(t) and f(s) are seen in tables Laplace transformation is profitable which mean in that many relationships and operation over the images can be correspond to simpler relationships and operations over the images.

1.1 Laplace integral transform

Integral transform are applied and helpful in the solution of partial differential equation. But, the choice of a particular transform to be used for the solution of a differential equation depends on the characteristic of the boundary condition of the equation. And the facility with which the transform f(p) can be inverted to provide -

$$f(s) = \int_{-\infty}^{\infty} e^{-st} d\alpha(t) \qquad (1.1,1)$$

Here, Let $\alpha(t)$ be of closely related variation in every finite interval. Particularly, if $\alpha(t)$ is an integral of a function $\phi(t)$, the integral (1.1, 1) becomes,

$$f(s) = \int_{-\infty}^{\infty} e^{-st} \phi(t) dt \qquad (1.1, 2)$$

Laplace transform or two sides Laplace transform by extending the limit of integration to be the entire real excess if that is done the common unilateral transform simply becomes a special case of integral transform

We say that $\alpha(t)$ is normalized in $(-\infty, \infty)$ if and only if $\alpha(0) = 0$ and

$$\alpha(t)=\frac{\alpha(t+)+\alpha(t-)}{2} \qquad (-\infty<t<\infty). \qquad (1.1, 3)$$

We observed a function normalized in interval $(0,\infty)$ and zero in $(-\infty,0)$ is not necessar[illegible] normalized in $(-\infty,\infty)$.

For the given values of s, whenever the integral (7.1,1) converges;

We found-

$$f(s)=\int_0^\infty e^{-st}d\alpha(t)+\int_0^\infty e^{st}d[-\alpha(-t)] \qquad (1.1, 4)$$

So, the study of the bilateral transform is lowered to the total of two unilateral transforms in one of [illegible] the variable s has been substituted by –s.

1.2 Region of Convergence

Clearly, the bilateral transform is the continuous analogue of the Laurent series.

$$F(z)=\sum_{n=-\infty}^{\infty}a_n z^n, \qquad (1.2, 1)$$

Where, f(z) be analytic in the ring shaped region surrounded by two concentric circles..

Axiom 7.2(a) If the following integral

$$f(s)=\int_{-\infty}^{\infty} e^{-st}d\alpha(t) \qquad (1.2, 2)$$

Converges for two points $s_1=\sigma_1+i\tau_1$ others $s_2=\sigma_2+i\tau_2(\sigma_1<\sigma_2)$,

Then, it is convergent in the vertical strip $\sigma_1<\sigma<\sigma_2$.

As an outcome applied in a trivial manner, the integral first converges for $s=\sigma_0+i\tau_1$ [illegible] converges for all $\sigma+i\tau$ for which $\sigma>\sigma_0$. For instance, the integral

$$\int_{-\infty}^{\infty}\frac{e^{-st}}{1+t^2}dt$$

Converges actually on the whole line $\sigma=0$ only. Lastly, the integral may have as its region [illegible] convergence certain parts of a vertical line. Thus if $\phi(t)=|t|^{-1/2}$, then the integral (1.2, 1) converge[illegible] the line $\sigma=0$ with an exception at the origin, and being divergent at all points off this line.

When the integral (1.2, 2) is convergent and divergent in the strip $\sigma_c'<\sigma<\sigma_c''$ [illegible] $\sigma>\sigma_c''$ and for $\sigma<\sigma_c'$ then each of the lines $\sigma=\sigma_c'$ and $\sigma=\sigma_c''$ is known as an axis [illegible] convergence and each of the members σ_c' and σ_c'' is an abscissa of convergence.

1.3 Integration by parts

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Here, we obtain an adequate & required condition for the integration by parts of a Laplace integral. We first explain the two theorems regarding the nature of $\alpha(t)$ at $+\infty$ and at $-\infty$.

Axiom 1.3(a) If the integral

$$s\,f(s)=\int_{-\infty}^{\infty}e^{-st}d\alpha(t) \qquad (1.3, 3)$$

Converges for $s=s_0=\gamma+i\delta$ with $\gamma>0$ then $\alpha(-\infty)$ exists and

$$\alpha(t)=o\left(e^{\gamma t}\right) \qquad (t\to\infty)$$

$$\alpha(t)-\alpha(-\infty)=o\left(e^{\gamma t}\right) \qquad (t\to-\infty)$$

This result follows from the decomposition (1.3, 3)

Axiom 1.3(b) If the integral (1.3, 3) is convergent for $s=s_0=\gamma+i\delta$ with $\gamma<0$ then $\alpha(\infty)$ exists and

$$\alpha(t)-\alpha(\infty)=o\left(e^{\gamma t}\right) \qquad (t\to\infty)$$

$$\alpha(t)=o\left(e^{\gamma t}\right) \qquad (t\to-\infty)$$

This proof is related to the Axiom 3.3(a). But, above mentioned theorems seems to be failed if $\gamma=0$

Axiom 1.3(c) If the integral (1.3, 3) is convergent for $s=s_0=\gamma+i\delta$ with $\gamma>0$ and if $\alpha(-\infty)=0$, then we have integral of converges which $\sigma>0$

$$f(s_0)=s_0\int_{-\infty}^{\infty}e^{-s_0t}\alpha(t)dt \qquad (1.3, 4)$$

For,

$$\int_0^{\infty}e^{-s_0t}d\alpha(t)=s_0\int_0^{\infty}e^{-s_0t}\alpha(t)dt-\alpha(0), \qquad (1.3, 5)$$

Converge with $\sigma<0$ then $\alpha(\infty)$ exist

$$\int_0^{\infty}e^{s_0t}d[-\alpha(-t)]=s_0\int_0^{\infty}e^{s_0t}\alpha(-t)dt+\alpha(0) \qquad (1.3, 6)$$

By adding equations (1.3, 5) & (1.3,6), we get equation (1.3,4).

Hence, a complete result!

Axiom 1.3(d) If the integral equation (1.3, 3) is convergent for $s=s_0=\gamma+i\delta$ with $\gamma<0$ and if $\alpha(\infty)=0$, then

$$f(s_0)=s_0\int_{-\infty}^{\infty}e^{-s_0t}\alpha(t)dt.$$

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The prior result by varying a variable t = - u. we have the restrictions following $\alpha(\infty)$ and $\alpha(-\infty)$ in these two theorems are not serious ones, and these numbers exist by virtue of Axiom 3.3(a) and 3.3(b). Thus, the finishing of the one desired can always be brought about by the addition of a required constant to the determining function. As such, this addition has no effect on the generating function. We must not let (1.3, 4) holds if $\gamma = 0$. Hence, if $\alpha(t)$ is the constant unity for positive and the constant zero for non-positive t, the above equation (1.3, 3) exists for all value of s and has the value unity. But, if $s_0 = 0 + i$

$$s_0 \int_{-\infty}^{\infty} e^{-s_0 t} \alpha(t) dt = i \int_0^{\infty} e^{-it} dt,$$

a divergent integral.

1.4 Determining function of Laplace integral

We place region of convergence of a bilateral Laplace integral from its determining function & too provide ourselves with the most useful of these. The others can easily be reached when needed by use of the decomposition (1.4,6).

Axiom 1.4(a) If

$$\overline{\lim_{t\to\infty}}\, t^{-1} \log|\alpha(t)| = k \neq 0$$

$$\underline{\lim_{t\to-\infty}}\, t^{-1} \log|\alpha(t)| = 1 \neq 0$$

with k < l, then the integral

$$\int_{-\infty}^{\infty} e^{-st} d\alpha(t) \qquad (1.4,7)$$

Converges for $k < \sigma < l$ and diverges for $\sigma < k$ and $\sigma > l$.

Clearly mentioned, if integral (1.4, 7) is convergent in a proper strip then, it is also convergent uniformly in any closed bounded region without touching the end areas of strip. Hence, in a stolz region there is also a uniform convergence following to a point of the end areas of the strip at which (1.4, 7) is convergent. So then, integral (1.4,7) is an analytic function .

$$f^{(k)}(s) = (-1)^k \int_{-\infty}^{\infty} e^{-st} d\alpha(t) \qquad (k = 0, 1, 2 ...)$$

f(s) is continuous at those boundary points of the strip where (1.4,7) is convergent.

We say that the integral (1.4,7) absolutely converges, if-

$$\int_{-\infty}^{\infty} e^{-\sigma t} |d\alpha(t)| = \int_{-\infty}^{\infty} c^{-\sigma t} du(t)$$

The above mentioned integral converges. Hence the function u (t) is the variation of $\sigma(x)$ in the interval $0 \leq x \leq t$. If t is non-negative, is zero if t is zero and is the non- positive of the variation of $\alpha(x)$ in the interval ...

interval $t \leq x \leq 0$ if t is non-positive. Clearly, u(t) is a increasing function in $(-\infty, \infty)$ which ends at the origin. Now, defining abscissas of absolute convergence σ'_a and σ''_a . As mentioned above definition of u(t) we get :

Axiom 1.4(b) If

$$\underline{\lim}_{t\to\infty} t^{-1} \log u(t) = k \neq 0$$

$$\underline{\lim}_{t\to-\infty} t^{-1} \log[-u(t)] = l \neq 0$$

And if k < l, then $\sigma'_a = k$ and $\sigma''_a = l$.

1.5 Laplace Inversion integral formulas

At very first, obtaining an inversion formula for the Laplace-Lebesgue integral (1.4,7).

Axiom 1.5(a) If $\phi(t)$ matches to L in every non-infinite interval, if the integral

$$f(s) = \int_{-\infty}^{\infty} e^{-st}\phi(t)dt \qquad (1.5, 8)$$

is convergent at $\sigma = c$, and if $\phi(t)$ is of bounded and neighbourhood of $t = t_0$, then

$$\lim_{T\to\infty} \frac{1}{2\pi i}\int_{e-iT}^{e+iT} f(s)e^{st_0}ds = \frac{\phi(t_0+)+\phi(t_0-)}{2} \qquad (1.5, 9)$$

Write (1.5,8) as the total of two integrals $f_1(s)$ and $f_2(s)$. According to the (0, ∞) and (−∞, 0), simultaneously, if t_0 is non-negative then, we get-

$$\lim_{T\to\infty} \frac{1}{2\pi i}\int_{c-iT}^{c+iT} f_1(s)e^{st_0}ds = \frac{\phi(t_0+)+\phi(t_0-)}{2}$$

$$\lim_{T\to\infty} \frac{1}{2\pi i}\int_{c-iT}^{c+iT} f_2(s)e^{st_0}ds = 0,$$

from which (1.5,9) follows at once.

If t_0 is zero, the same theorem gives

$$\lim_{T\to\infty} \frac{1}{2\pi i}\int_{c-iT}^{c+iT} f_1(s)ds = \frac{\phi(0+)}{2}$$

$$\lim_{T\to\infty} \frac{1}{2\pi i}\int_{c-iT}^{c+iT} f_2(s)ds = \frac{\phi(0-)}{2},$$

Thus, on adding we found the desired result.

Axiom 1.5(b) If $\alpha(t)$ is a normalized function of bounded variation in each of all non-infinite interval and if the integral-

$$f(s)=\int_{-\infty}^{\infty}e^{-st}d\alpha(t)$$

is convergent $k<\sigma<1$, then for all t, we have-

$$\lim_{T\to\infty}\frac{1}{2\pi i}\int_{c-iT}^{c+iT}\frac{f(s)}{s}e^{st}ds=\begin{cases}\alpha(t)-\alpha(-\infty) & (c>0,k<c<1)\\ \alpha(t)-\alpha(\infty) & (c<0,k<c<1)\end{cases} \qquad (1.5,10)$$

f(s) as the sum of both integrals i.e. $f_1(s)$ and $f_2(s)$ considering in the prior result proof. If, t is non-negative; then we have-

$$\lim_{T\to\infty}\frac{1}{2\pi i}\int_{c-iT}^{c+iT}\frac{f_1(s)}{s}e^{st}ds=\alpha(t) \qquad (c>0) \quad (1.5,11)$$

$$=\alpha(t)-\alpha(\infty) \qquad (c<0)$$

$$\lim_{T\to\infty}\frac{1}{2\pi i}\int_{c-iT}^{c+iT}\frac{f_2(s)}{s}e^{st}ds=-\alpha(-\infty) \qquad (c>0) \quad (1.5,12)$$

$$=0 \qquad (c<0).$$

Sum of two equation (1.5,11) and (1.5,12) to get equation (1.5,10).The value of t is non-positive or obtains alternate changes in variables. Lastly, when t=o.

$$\lim_{T\to\infty}\frac{1}{2\pi i}\int_{c-iT}^{c+iT}\frac{f_1(s)}{s}ds=\frac{\alpha(0+)}{2} \qquad (c>0) \qquad (1.5,13)$$

$$(c<0)$$

$$\lim_{T\to\infty}\frac{1}{2\pi i}\int_{c-iT}^{c+iT}\frac{f_2(s)}{s}ts=\frac{\alpha(0-)}{2}-\alpha(-\infty) \qquad (c>0) \qquad (1.5,14)$$

$$=\frac{\alpha(0-)}{2} \qquad (c<0).$$

Similarly; equation (1.5,13) and (1.5,14) with t = 0 since $[\alpha(0+)+\alpha(0-)]/2$ is equal to $\alpha(0)$ by our definition of normalization.

For example of this axiom f(s) = 1 so that $\alpha(t)$ may be taken $-1/2$ for $-\infty<t<0$ and $+1/2$ for $0<t<\infty$. It is easily visualize by Cauchy's theorem,.

$$\lim_{T\to\infty}\frac{1}{2\pi i}\int_{c-iT}^{c+iT}\frac{e^{st}}{s}ds=1=\alpha(t)-\alpha(-\infty) \qquad (t>0)$$

$$=\frac{1}{2}=\frac{\alpha(0+)+\alpha(0-)}{2}-\alpha(-\infty) \qquad (t=0)$$

$$=0=\alpha(t)-\alpha(-\infty) \qquad (t<0).$$

Taking non-negative value of c, so that the theorem satisfies in this special case. Correspondingly, we get

$$\lim_{T\to\infty}\frac{1}{2\pi i}\int_{-1-iT}^{-1+iT}\frac{e^{st}}{s}ds=-1=\alpha(t)-\alpha(\infty) \qquad (t<0)$$

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$$= -\frac{1}{2} = \frac{\alpha(0+)+\alpha(0-)}{2} - \alpha(\infty) \qquad (t = 0)$$

$$= 0 = \alpha(t) - \alpha(\infty) \qquad (t > 0),$$

The above theorem is again proved for non-positive value of constant c.

1.6 Function of a Laplace integral

The inversion formulas formed in the last section enable us to create a discussion on the uniqueness of the representation of a function by a Laplace integral. This theorem is for the transform, so that the distinct approach is enabled and wanted as per the requirement.

Axiom 1.6 (a) If $\alpha_1(t)$ and $\alpha_2(t)$ are two normalized functions of bounded variation in every finite interval such that-

$$\int_{-\infty}^{\infty} e^{-st} d\alpha_1(t) = \int_{-\infty}^{\infty} e^{-st} d\alpha_2(t) \qquad (1.6, 15)$$

Now, taking a common narrow piece of convergence $k < \sigma < 1$, then $\alpha_1(t) = \alpha_2(t)$ for all value of t.

Using Axiom 1.5(b),

$$\alpha_1(t) - \alpha_1(-\infty) = \alpha_2(t) - \alpha_2(-\infty) \; (-\infty < t < \infty) \qquad (1.6, 16)$$

When interval (k, l) initiates all points of the non-negative axis, &

$$\alpha_1(t) - \alpha_1(\infty) = \alpha_2(t) - \alpha_2(\infty) \; (-\infty < t < \infty) \qquad (1.6, 17)$$

thus, the interval (k, l) enables all points of the non-positive axis. As $\alpha_1(0) = \alpha_2(0) = 0$ it follows that $\alpha_1(-\infty) = \alpha_2(-\infty)$ in (1.6, 30) or $\alpha_1(\infty) = \alpha_2(\infty)$ in (1.6, 17). This proves the result.

Axiom 1.6(b) If $\phi_1(t)$ and $\phi_2(t)$ are of class L in every non-infinite interval, then

$$\int_{-\infty}^{\infty} e^{-st}\phi_1(t)dt = \int_{-\infty}^{\infty} e^{-st}\phi_2(t)dt \qquad (1.6, 18)$$

in a common strip of convergence $k < \sigma < l$, then $\phi_1(t)$ is equal to $\phi_2(t)$ for almost all t.

By equation (1.6, 29) and (1.6,32), if-

$$\alpha_1(t) = \int_0^t \phi_1(u)du \qquad (-\infty < t < \infty)$$

$$\alpha_2(t) = \int_0^t \phi_2(u)du \qquad (-\infty < t < \infty)$$

we have equations (1.6,16) or (1.6,17) $\alpha_1'(t) = \alpha_2'(t)$ at all the points where there is an existence of these derivatives. But for almost all t-

$$\alpha_1'(t) = \phi_1(t)$$

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$$\alpha_2'(t) = \phi_2(t),$$

Thus, the result is established.

Conclusion

Laplace integral transform, integration by parts, determining function of Laplace integral, inversion integral formulas. This chapter also consist function of a Laplace integral, chapter also shows resultant of indefinite integrals, ilterates of an explicit expression.

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The Stieltjes Moment Problem Related to the Hamberger Problem

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Abstract :- This present paper deals with the Stieltjes moment problem related to the hamberger problem. A significant and required condition that there should exist an increasing function $\alpha(t)$ such as-

$$\mu_n = \int_0^{\infty} t^n d\alpha(t) \qquad (n = 0, 1, 2, \ldots),$$

equations should have an increasing solution $\alpha(t)$ with infinitely many points of non-decrease and equations should have an increasing solution $\alpha(t)$ with a finite number of points of non-decrease .

Keywords :- Stieltjes Moment Problem, Hamberger problem, Hausdroff's moment problem, Increasing function ,Distribution function.

1 Introduction :- The moment problem on [0,1) is referred to as the Hausdroff's moment problem and the moment problem on R is called the Hamburger moment problem and thus $[0,\infty)$ is called the Stieltjes moment problem[1]. The Stieltjes Moment Problem seeks for a non decreasing positive distribution function on the semi-axis so that its moment match a given infinite sequence of positive real numbers [2-4].

Hausdroff 's moment problem often known as moment problem is given as below-

$$\{\mu_n\}_0^{\infty} : \mu_0, \mu_1, \mu_2, \ldots; \qquad (1,1)$$

There are other forms of existence to predict function $\alpha(t)$ of bounded variation in interval (0,1) for example -

$$\mu_n = \int_0^1 t^n d\alpha(t) \qquad (n = 0, 1, 2,\ldots) \qquad (1,2)$$

Hence, this is corollary known as moment sequence. But, it has been noticed that every sequence (1,1) has them form (1,2) since (1,2) implies that-

$$|\mu_n| \leq V[\alpha(t)]_0^1$$

the quantity on the right being the variation of $\alpha(t)$ on the interval (0, 1). Thus, in this way its sequence is confined. It was F. Hausdorff [1921a] to determine essential situation of sequence to be moment sequence. But, given that representation (1,2) if $\alpha(t)$ is a normalized function of corollary differences.

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$$\alpha(0)=0, \quad \alpha(t)=\frac{\alpha(t+)+\alpha(t-)}{2} \qquad (0<t<1).$$

As normalization of the function $\alpha(t)$ does not varies the value of the integral (1,2) so, we can take over the loss with any disturbance to general prospect $\alpha(t)$ which is normalized. Infact, the entire discussion throw light upon it without any obstacle.

Equations (1,2) may be admired as an inter-changeability of the function $\alpha(t)$ into the sequence $\{\mu_n\}$. This inter-changeability is closely attached to the Laplace transform, is in fact the discrete analogue of the latter. For, if we substitute the integer n by the variable s in (1,2) and then make the change of variable $t = e^{-u}$, we obtain-

$$\mu_s = \int_0^\infty e^{-su} d[-\alpha(e^{-u})].$$

Hence, it is perfectly observed that enough importance relays on Laplace transform by a solution of the moment problem.

2 Material and Methods

The Hamburger and Stieltjes problems for the case in which $\alpha(t)$ is of closely related variation on the appropriate infinite interval. R.P. Boas [1939] has observed that in this case there is hardly any problem, as every sequence leads to a soluble Stieltjes or Hamburger problem if we consider any function of closely related variation as a solution[2].

We provide the proof of Boas. which will of course be enough to initialize the Stieltjes case.

The equations

$$\mu_n = \int_0^\infty t^n d\alpha(t) \qquad (n = 0, 1, 2, ...)$$

For ever have a solution $\alpha(t)$ of closely related variation for which

$$\int_0^\infty |d\alpha(t)| < \infty .$$

We formed the other two sequences $\{\lambda_n\}_0^\infty, \{\nu_n\}_0^\infty$, such as;

$$\mu_n = \lambda_n - \nu_n$$

$$\lambda_n = \int_0^\infty t^n d\beta(t)$$

$$\nu_n = \int_0^\infty t^n d\gamma(t) \qquad (n = 0, 1, 2, ...),$$

3 Result and discussion

3.1 The Stieltjes Moment Problem Related to the Humberger Problem

The Stieltjes problem can also be applied as a special case of the Hamburger problem[7,16].

Axiom 3.1(a) A significant and required condition that there should exist an increasing function $\alpha(t)$ such as-

$$\mu_n = \int_0^\infty t^n d\alpha(t) \qquad (n = 0, 1, 2, ...), \qquad (3.1,1)$$

The integrals are all converging, thus; the sequences $\{\mu_n\}_0^\infty$ and $\{\mu_n\}_1^\infty$ should be non-negative, or that the quadratic forms

$$\sum_{i=0}^{n}\sum_{j=0}^{n}\mu_{i+j}\xi_i\xi_j \qquad (n = 0, 1, 2, ...) \qquad (3.1,2)$$

$$\sum_{i=0}^{n}\sum_{j=0}^{n}\mu_{i+j+1}\xi_i\xi_j \qquad (n = 0, 1, 2, ...) \qquad (3.1,3)$$

should be non-negative (definite or semi-definite).

According to Axiom - The equivalence of the two forms of the condition is apparent. We prove the result in the latter form involving quadratic forms. For the requirement, Let the sequence $\{\mu_n\}_0^\infty$ have the form (3.1,1). since we may regard $\alpha(t)$ as constant in the interval $(-\infty, 0)$. Proceeding further,

$$\mu_{n+1} = \int_0^\infty t^n d\beta(t) \qquad (n = 0, 1, ...)$$

$$\beta(t) = \int_0^t u d\alpha(u) \qquad (t \geq 0).$$

As $\beta(t)$ is also an increasing so, Axiom(a) A necessary and required condition that there should be an existence of at least one increasing function $\alpha(t)$ in a way as

$$\mu_n = \int_{-\infty}^\infty t^n d\alpha(t) \qquad (n = 0, 1, 2, ...),$$

all the integrals converging, is that the sequence $\{\mu_n\}_0^\infty$ should be non-negative. And Axiom(b)A significantly required and necessary condition that the sequence $\{\mu_n\}_0^\infty$ should be non-negative definite (semi-definite) is that the quadratic forms

$$\sum_{i=0}^{n}\sum_{j=0}^{n}\mu_{i+j}\xi_i\xi_j \qquad (n = 0, 1, 2, ...)$$

could be non-negative definite (semi-definite).

Axiom (a) and (b) show that the forms (3.1,3) are also non-negative.

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Contradictory, suppose the forms (3.1,2) and (3.1,3) be non-negative and then considering the new sequence $\{\nu_n\}_0^\infty$ where,

$$\nu_{2n} = \mu_n \qquad (n = 0, 1, ...)$$

$$\nu_{2n+1} = 0 \qquad (n = 0, 1, ...).$$

If n = odd,

$$\sum_{i=0}^{n}\sum_{j=0}^{n}\nu_{i+j}\xi_i\xi_j = \sum_{i=0}^{\frac{n-1}{2}}\sum_{j=0}^{\frac{n-1}{2}}\mu_{i+j}\xi_{2i}\xi_{2j}\sum_{i=0}^{\frac{n-1}{2}}\sum_{j=0}^{\frac{n-1}{2}}\mu_{i+j+1}\xi_{2i+1}\xi_{2j+1},$$

and now if n = even;

$$\sum_{i=0}^{n}\sum_{j=0}^{n}\nu_{i+j}\xi_i\xi_j = \sum_{i=0}^{\frac{n}{2}}\sum_{j=0}^{\frac{n}{2}}\mu_{i+j}\xi_{2i}\xi_{2j}\sum_{i=0}^{\frac{n}{2}-1}\sum_{j=0}^{\frac{n}{2}-1}\mu_{i+j+1}\xi_{2i+1}\xi_{2j+1},$$

This proves that the sequence $\{\nu_n\}_0^\infty$ is non-negative, and thus by Axiom (a) it stated that there lies an existence of an increasing function $\beta(t)$ in such a way-

$$\nu_n = \int_{-\infty}^{\infty} t^n d\beta(t) \qquad (n = 0, 1, 2, ...),$$

Otherwise,

$$\mu_n = \int_{-\infty}^{\infty} t^{2n} d\beta(t) \qquad (n = 0, 1, 2, ...)$$

$$0 = \int_{-\infty}^{\infty} t^{2n+1} d\beta(t) \qquad (n = 0, 1, 2, ...). \qquad (3.1,4)$$

Set

$$\gamma(t) = \frac{\beta(t) - \beta(-t)}{2} \qquad (-\infty < t < \infty).$$

Here, the function is odd which also satisfy the equations (3.1,4). It is increasing.

Set $\alpha(t) = 2\gamma(t^{1/2})(t \geq 0)$. So, Then-

$$\mu_n = \int_{-\infty}^{\infty} t^{2n} d\gamma(t) = \int_0^{\infty} t^{2n} d\gamma(t) - \int_0^{\infty} t^{2n} d\gamma(-t)$$

by an obvious change of variables. But since $\gamma(t)$ is odd, this gives

$$\mu_n = 2\int_0^{\infty} t^{2n} d\gamma(t) \qquad (n = 0, 1, 2, ...)$$

$$= \int_0^{\infty} t^n d[2\gamma(t^{1/2})]$$

$$= \int_0^{\infty} t^n d\alpha(t).$$

Since, $\alpha(t)$ is increasing in the interval $0 \leq t < \infty$, we reached the expected outcome.

Clearly stated that we also have the following results.

Axiom 3.1(b) A significant and required condition that equations (3.1,1) should have an increasing solution $\alpha(t)$ with infinitely many points of non-decrease is that the forms (3.1,2) and (3.1,3) should all be non-negative definite or that the zero must be

$$\mu_0, \quad \begin{vmatrix} \mu_0 & \mu_1 \\ \mu_1 & \mu_2 \end{vmatrix}, \quad \begin{vmatrix} \mu_0 & \mu_1 & \mu_2 \\ \mu_1 & \mu_2 & \mu_3 \\ \mu_2 & \mu_3 & \mu_4 \end{vmatrix}, \ldots \qquad (3.1,5)$$

$$\mu_1, \quad \begin{vmatrix} \mu_1 & \mu_2 \\ \mu_2 & \mu_3 \end{vmatrix}, \quad \begin{vmatrix} \mu_1 & \mu_2 & \mu_3 \\ \mu_2 & \mu_3 & \mu_4 \\ \mu_3 & \mu_4 & \mu_5 \end{vmatrix}, \ldots$$

lesser than all the determinants.

Axiom 3.1(c) A required and significant condition that equations (3.1,1) should have an increasing solution $\alpha(t)$ with a finite number of points of non-decrease is that the forms (3.1,2) and (3.1,3) should all be non-negative & at least one of them being non-negative semi-definite.

We observe that it is not enough that the determinants (3.1,4) should be all positive.

3.2 The Humburger Problem-

According to the Hausdorff problem one may expect that it could be expectable to cope with the Hamburger and Stieltjes problems for the case in which $\alpha(t)$ is of closely related variation on the appropriate infinite interval. R.P. Boas [1939] has observed that in this case there is hardly any problem, as every sequence leads to a soluble Stieltjes or Hamburger problem if we consider any function of closely related variation as a solution [13].

We provide the proof of Boas. which will of course be enough to initialize the Stieltjes case[2-6].

Axiom 3.2(a) The equations

$$\mu_n = \int_0^\infty t^n d\alpha(t) \qquad (n = 0, 1, 2, \ldots)$$

For ever have a solution $\alpha(t)$ of closely related variation for which

$$\int_0^\infty |d\alpha(t)| < \infty.$$

We formed the other two sequences $\{\lambda_n\}_0^\infty, \{\nu_n\}_0^\infty$, such as;

$$\mu_n = \lambda_n - \nu_n \qquad (3.2, 6)$$

$$\lambda_n = \int_0^\infty t^n d\beta(t) \qquad (3.2, 7)$$

$$\nu_n = \int_0^\infty t^n d\gamma(t) \qquad (n = 0, 1, 2, \ldots), \qquad (6.2, 8)$$

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Now, here $\beta(t)$ and $\gamma(t)$ are closely related increasing functions. First choose $\lambda_0, \lambda_1, v_0, v_1$ as any non-negative numeric satisfying (3.2, 6). Proceeding by induction, Let, λ_k, v_k for k = 0, 1, 2, ...,2n – 1 so that (3.2,6) holds and so that the determinants

$$[\lambda_0, \lambda_1, \ldots, \lambda_{2k}] = \begin{vmatrix} \lambda_0 & \lambda_1 & \cdots & \lambda_k \\ \lambda_1 & \lambda_2 & \cdots & \lambda_{k+1} \\ \cdot & \cdot & \cdots & \cdot \\ \lambda_k & \lambda_{k+1} & \cdots & \lambda_{2k} \end{vmatrix} \tag{3.2, 9}$$

$$[\lambda_1, \lambda_2, \ldots, \lambda_{2k+1}] = \begin{vmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_{k+1} \\ \lambda_2 & \lambda_3 & \cdots & \lambda_{k+2} \\ \cdot & \cdot & \cdots & \cdot \\ \lambda_{k+1} & \lambda_{k+2} & \cdots & \lambda_{2k+1} \end{vmatrix},$$

$$[v_0, v_1, \ldots, v_{2k}], \quad [v_1, v_2, \ldots, v_{2k+1}] \tag{3.2,10}$$

are non-negative for k = 0, 1, ...n – 1. Thus, we explain &define $\lambda_{2n}, v_{2n}, \lambda_{2n+1}, v_{2n+1}$. We have undetermined λ_{2n}

$$[\lambda_0, \lambda_1, \ldots, \lambda_{2n}] = \lambda_{2n}[\lambda_0, \lambda_1, \ldots, \lambda_{2n-2}] + P, \tag{3.2,11}$$

Here, P is a polynomial in $\lambda_0, \lambda_1, \ldots, \lambda_{2n-1}$; and similarly for $[v_0, v_1, \ldots, v_{2n}]$. Since zero is smaller than $[\lambda_0, \lambda_1, \ldots, \lambda_{2n-2}]$ and $[v_0, v_1, \ldots, v_{2n-2}]$ so, we can choose λ_{2n} and v_{2n} non-negative and so large that $\lambda_{2n} - v_{2n} = \mu_{2n}$ as-

$$[\lambda_0, \lambda_1, \ldots, \lambda_{2n}] > 0, \quad [v_0, v_1, \ldots, v_{2n}] > 0$$

It has been observed that (3.2,11) holds with all subscripts increased by unity, P now being a polynomial in $\lambda_1, \lambda_2, \ldots, \lambda_{2n}$, a similar equation holding for the v_k. With λ_{2n} and v_{2n} now determined we proceed exactly as above to determine λ_{2n+1} and v_{2n+1}. Thus, the induction is completed. Following Axiom 3.1(b) if the determinants (3.2,9) and (3.2,10) are non-negative for k = 0, 1, 2, ... equations (3.2, 7) and (3.2, 8) have closely related solutions $\beta(t)$ and $\gamma(t)$ respectively, so that when $\alpha(t)$ is explained & defined as $\beta(t) - \gamma(t)$ our proof is proved.

Stieltjes is mentioned to prove there exists a function, not a constant, all the moments of which are zero. It too follows from Axiom 3.2(a). For, by this result, there is an existence of a non-constant function $\alpha(t)$ such as-

$$\int_0^\infty t^n d\alpha(t) = 1 \qquad (n = 1)$$

$$(n = 0, 2, 3, 4, ...)$$

$$\int_0^\infty |d\alpha(t)| < \infty .$$

Putting;

$$\beta(t) = \alpha(t^{1/2}),$$

we form-

$$\int_0^\infty t^n d\beta(t) = \int_0^\infty t^{2n} d\alpha(t) = 0 \qquad (n = 0, 2, 3, 4, ...).$$

The function $\beta(t)$ is the required example .

Conclusion

In this paper the applications of moment problem to various area deals with the stieltjes moment problem related to the hamburger problem.Asignificant and required condition that there should exist an increasing function $\alpha(t)$ such as-

$$\mu_n = \int_0^\infty t^n d\alpha(t) \qquad (n = 0, 1, 2, ...),$$

equations should have an increasing solution $\alpha(t)$ with infinitely many points of non-decrease and equations should have an increasing solution $\alpha(t)$ with a finite number of points of non-decrease .

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A Significant Condition for Solving Moment Problem

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ABSTRACT : This present paper deals with the moment problem.The Hamburger and Stieltjes problems for the case in which $\alpha(t)$ is of closely related variation on the appropriate infinite interval. R.P. Boas [1939] has observed that in this case there is hardly any problem, as every sequence leads to a soluble Stieltjes or Hamburger problem if we consider any function of closely related variation as a solution [2].condition for solving moment problem and determining and non-determining solution.

KEYWORDS : Moment Problem, Stieltjes problem, Hamburger moment problem, Inequalities, Moment sequence, Laplace transform.

I. INTRODUCTION

1.1 Moment Problem

As per engineering mechanicals tools and technique, When a force of certain magnitude is applied on a particle then moment is produce, mathematically, this is measured as the product of force applied and the effective distance of the shape and size of the particle or substance taken for the pupose. Simple motto behind explaining this definition is that moment is govern by two variables and so there should be limitation of exact two variables in case of moment based problems. This fact is an eyes opener for those who consider only one variable and and talk about existence of a Moment Problem only.

II RELATED WORK

When μ is a positive measure with moments $(s_n)_{n>=0}$ we say that μ is a solution to the moment problem. If the solution to the moment problem is unique, the moment problem is called determinate.otherwise the moment problem is said to be indeterminate [1]. The moment problem on [0,1) is referred to as the Hausdroff's moment problem and the moment problem on R is called the Hamburger moment problem and thus [0,∞) is called the Stieltjes moment problem [12].

Statement of the moment problem is given as below-

$$\{\mu_n\}_0^\infty : \mu_0, \mu_1, \mu_2, \ldots; \qquad (1.1,1)$$

There are other forms of existence to predict function $\alpha(t)$ of bounded variation in interval (0,1) for example -

$$\mu_n = \int_0^1 t^n d\alpha(t) \qquad (n = 0, 1, 2, \ldots) \qquad (1.1,2)$$

Hence, this is corollary known as moment sequence. But, it has been noticed that every sequence (1.1,1) has them form (1.1,2) since (1.1,2) implies that-

$$|\mu_n| \leq V[\alpha(t)]_0^1$$

the quantity on the right being the variation of $\alpha(t)$ on the interval (0, 1). Thus, in this way its sequence is confined. It was F. Hausdorff [1921a] to determine essential situation of sequence to be moment sequence. But, given that representation (1.1,2) if $\alpha(t)$ is a normalized function of corollary differences.

$$\alpha(0) = 0, \quad \alpha(t) = \frac{\alpha(t+) + \alpha(t-)}{2} \qquad (0 < t < 1).$$

As normalization of the function $\alpha(t)$ does not varies the value of the integral (1.1,2) so, we can take over the loss with any disturbance to general prospect $\alpha(t)$ which is normalized. Infact, the entire discussion throw light upon it without any obstacle.

Equations (1.1,2) may be admired as an inter-changeability of the function $\alpha(t)$ into the sequence $\{\mu_n\}$. This inter-changeability is closely attached to the Laplace transform, is in fact the discrete analogue of the latter. For, if we substitute the integer n by the variable s in (1.1,2) and then make the change of variable $t = e^{-u}$, we obtain-

$$\mu_s = \int_0^\infty e^{-su} d[-\alpha(e^{-u})].$$

Hence, it is perfectly observed that enough importance relays on 'Laplace transform' by a solution of the moment problem.

III. METHODOLOGY

2.1 The Hamburger Problem

The Hamburger and Stieltjes problems for the case in which $\alpha(t)$ is of closely related variation on the appropriate infinite interval [13]. R.P. Boas [1939] has observed that in this case there is hardly any problem, as every sequence leads to a soluble Stieltjes or Hamburger problem if we consider any function of closely related variation as a solution. We provide the proof of Boas. which will of course be enough to initialize the Stieltjes case. Axiom 2.1(a) The equations

$$\mu_n = \int_0^\infty t^n d\alpha(t) \qquad (n = 0, 1, 2, ...)$$

For ever have a solution $\alpha(t)$ of closely related variation for which

$$\int_0^\infty |d\alpha(t)| < \infty .$$

We formed the other two sequences $\{\lambda_n\}_0^\infty, \{\nu_n\}_0^\infty$, such as;

$$\mu_n = \lambda_n - \nu_n \qquad (2.1,1)$$

$$\lambda_n = \int_0^\infty t^n d\beta(t) \qquad (2.1,2)$$

$$\nu_n = \int_0^\infty t^n d\gamma(t) \qquad (n = 0, 1, 2, ...), \qquad (2.1,3)$$

Now, here $\beta(t)$ and $\gamma(t)$ are closely related increasing functions. First choose $\lambda_0, \lambda_1, \nu_0, \nu_1$ as any non-negative numeric satisfying (2.1,1). Proceeding by induction, Let, λ_k, ν_k for k = 0, 1, 2, ...,2n – 1 so that (2.1,1) holds and so that the determinants

$$[\lambda_0, \lambda_1, ..., \lambda_{2k}] = \begin{vmatrix} \lambda_0 & \lambda_1 & \cdots & \lambda_k \\ \lambda_1 & \lambda_2 & \cdots & \lambda_{k+1} \\ . & . & \cdots & . \\ \lambda_k & \lambda_{k+1} & \cdots & \lambda_{2k} \end{vmatrix} \qquad (2.1,4)$$

$$[\lambda_1, \lambda_2, ..., \lambda_{2k+1}] = \begin{vmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_{k+1} \\ \lambda_2 & \lambda_3 & \cdots & \lambda_{k+2} \\ . & . & \cdots & . \\ \lambda_{k+1} & \lambda_{k+2} & \cdots & \lambda_{2k+1} \end{vmatrix},$$

$$[\nu_0, \nu_1, ..., \nu_{2k}], \quad [\nu_1, \nu_2, ..., \nu_{2k+1}] \qquad (2.1,5)$$

are non-negative for k = 0, 1, ...n – 1. Thus, we explain & define $\lambda_{2n}, \nu_{2n}, \lambda_{2n+1}, \nu_{2n+1}$. We have undetermined λ_{2n}

$$[\lambda_0, \lambda_1, ..., \lambda_{2n}] = \lambda_{2n}[\lambda_0, \lambda_1, ..., \lambda_{2n-2}] + P, \qquad (2.1,6)$$

Here, P is a polynomial in $\lambda_0, \lambda_1, \ldots, \lambda_{2n-1}$; and similarly for $[\nu_0, \nu_1, \ldots, \nu_{2n}]$. Since zero is smaller than $[\lambda_0, \lambda_1, \ldots, \lambda_{2n-2}]$ and $[\nu_0, \nu_1, \ldots, \nu_{2n-2}]$ so, we can choose λ_{2n} and ν_{2n} non-negative and so large that $\lambda_{2n} - \nu_{2n} = \mu_{2n}$ as-

$$[\lambda_0, \lambda_1, \ldots, \lambda_{2n}] > 0, \quad [\nu_0, \nu_1, \ldots, \nu_{2n}] > 0$$

It has been observed that (2.1,6) holds with all subscripts increased by unity, P now being a polynomial in $\lambda_1, \lambda_2, \ldots, \lambda_{2n}$, a similar equation holding for the ν_k. With λ_{2n} and ν_{2n} now determined we proceed exactly as above to determine λ_{2n+1} and ν_{2n+1}. Thus, the induction is completed. Following Axiom 2.1(b) if the determinants (2.1,4) and (2.1,5) are non-negative for k = 0, 1, 2, ... equations (2.1,2) and (2.1,3) have closely related solutions $\beta(t)$ and $\gamma(t)$ respectively, so that when $\alpha(t)$ is explained & defined as $\beta(t) - \gamma(t)$ our proof is proved.

Stieltjes is mentioned to prove there exists a function, not a constant, all the moments of which are zero. It too follows from Axiom 2.1(a). For, by this result, there is an existence of a non-constant function $\alpha(t)$ such as-

$$\int_0^\infty t^n d\alpha(t) = 1 \qquad (n = 1)$$

$$(n = 0, 2, 3, 4, \ldots)$$

$$\int_0^\infty |d\alpha(t)| < \infty .$$

Putting;

$$\beta(t) = \alpha(t^{1/2}),$$

we form-

$$\int_0^\infty t^n d\beta(t) = \int_0^\infty t^{2n} d\alpha(t) = 0 \qquad (n = 0, 2, 3, 4, \ldots).$$

The function $\beta(t)$ is the required example.

IV. RESULTS AND DISCUSSION

3.1 The condition for solving moment problem

Boas showed that any sequence which increases sufficiently rapidly leads to a soluble Stieltjes problem [with increasing $\alpha(t)$] by a slight modification of the method employed. More precisely, the outcome is

Axiom 3.1(a) If

$$\mu_0 \geq 1, \quad \mu_n \geq (n\mu_{n-1})^n \qquad (n = 1, 2, \ldots) \qquad (3.1,7)$$

So, the equations

$$\mu_n = \int_0^\infty t^n d\alpha(t) \qquad (n = 0, 1, 2, \ldots)$$

have an increasing solution $\alpha(t)$

For example- Given sequence satisfying (2.1, 1) is $\mu_0 = 1, \mu_n = n^{n^n}$ for n = 1, 2, ... as considered

$$[\mu_0, \mu_1, \ldots, \mu_{2n}] = \mu_{2n}[\mu_0, \mu_2, \ldots, \mu_{2n-2}] + \sum_{k=n}^{2n-1} \pm \mu_k D_k, \qquad (3.1,8)$$

Where, the D_k are n-rowed minors of $[\mu_0, \mu_1, \ldots, \mu_{2n}]$ not containing μ_{2n}. Similarly

$$[\mu_1, \mu_2, \ldots, \mu_{2n+1}] = \mu_{2n+1}[\mu_1, \mu_2, \ldots, \mu_{2n-1}] + \sum_{k=n+1}^{2n} \pm \mu_k D'_k \qquad (3.1,9)$$

The D'_k are again n-rowed minors of $[\mu_1, \mu_2, \ldots, \mu_{2n+1}]$ not containing μ_{2n+1}.

Let, $k \leq m-1$ we have given that-

$$[\mu_0, \mu_1, \ldots, \mu_{2k}] \geq 1, \qquad [\mu_1, \mu_2, \ldots, \mu_{2k+1}] \geq 1 \qquad (3.1,10)$$

To prove the same inequalities for k = m, using (3.1,7) we observe that the sequence $\{\mu_n\}_0^\infty$ is an increasing function and thus, unity is greater than then no element of the sequence. So,

$$\mu_n > 2\left(\frac{n}{2}\right)^{\frac{n+4}{4}} (\mu_{n-1})^{\frac{n+2}{2}} \qquad (n = 2, 3, ...).$$

Particularly-

$$\mu_{2n} > 2n^{\frac{n+4}{4}} (\mu_{2n-1})^{n+1} \geqq 1 + n^{\frac{n+2}{2}} (\mu_{2n-1})^{n+1} \qquad (n = 2, 3, ...)$$

$$\mu_{2n+1} > 2n^{\frac{n+4}{4}} (\mu_{2n})^{n+1} \geqq 1 + n^{\frac{n+2}{2}} (\mu_{2n})^{n+1} \qquad (n = 2, 3, ...) \qquad (3.1,11)$$

The elements of D_k are smaller than μ_{2m-1} and those of D'_k are also smaller than μ_{2m} where k ranges over the integer mentioned in the summations (3.1, 8) and (3.1,9). Thus, by Hadamard's upper bound for a determinant- we get,

$$|D_k| \leqq m^{\frac{m}{2}} (\mu_{2m-1})^m \qquad (k = m, m+1, ..., 2m-1)$$

$$|D'_k| \leqq m^{\frac{m}{2}} (\mu_{2m})^m \qquad (k = m+1, m+2, ..., 2m).$$

Thus;

$$\left|\sum_{k=m}^{2m-1} \pm \mu_k D_k\right| \leqq m(\mu_{2m-1}) m^{\frac{m}{2}} (\mu_{2m-1})^m$$

$$\left|\sum_{k=m}^{2m} \pm \mu_k D'_k\right| \leqq m(\mu_{2m}) m^{\frac{m}{2}} (\mu_{2m})^m,$$

so that by (6.23) and (6.24)

$$[\mu_0, \mu_1, \ldots, \mu_{2m}] \geqq \mu_{2m} - m^{\frac{m+2}{2}} (\mu_{2m-1})^{m+1}$$

$$[\mu_1, \mu_2, \ldots, \mu_{2m+1}] \geqq \mu_{2m+1} - m^{\frac{m+2}{2}} (\mu_{2m})^{m+1}$$

As per our observation by (3.1,11) we concluded that (3.1,10) is established for k = m. Through induction (3.1,10) now holds for all k, and by the Stieltjes moment problem similar to the moments (3.1,7) has an increasing solution $\alpha(t)$. Hence, proved!.

3.2 Determining and non-determining solution

Boas showed that any sequence of sufficiently continuous strong growth leads to a Stieltjes problem which has more than one increasing solution by making use of the last outcome[2-6].

Axiom 3.2(a) If

$$\lambda_0 \geqq 1$$

$$\lambda_2 \geqq (2\lambda_1 + 2)^2$$

$$\lambda_n \geqq (n\lambda_{n-1})^n \qquad (n = 1, 3, 4, 5, ...)$$

$$\mu_n = \lambda_{2n} \qquad (n = 0, 1, 2, ...),$$

So, then there are at least two necessary and significant different increasing functions $\alpha(t)$ such as-

$$\mu_n = \int_0^\infty t^n d\alpha(t) \qquad (n = 0, 1, 2, ...). \quad (3.2,12)$$

There exists a function $\beta(t)$ which is non-negative, increasing, such as given below;

$$\lambda_n = \int_0^\infty t^n d\beta(t^{1/2}) \qquad (n = 0, 1, ...),$$

Thus,

$$\mu_n = \int_0^\infty t^n d\beta(t^{1/2}) \qquad (n = 0, 1, 2, ...).$$

Next, we suppose $\{\nu_n\}_0^\infty$ be a sequence which is corresponding to the sequence $\{\lambda_n\}_0^\infty$ with an exception $\nu_1 = \lambda_1 + 1$.

$$\nu_n \geq (n\nu_{n-1})^n \qquad (n = 1, 2, 3, ...).$$

Obviously, if n is neither 1 nor 2. Then;

$$\lambda_1 + 1 = \nu_1 \geq \nu_0 = \lambda_0$$

$$\nu_2 = \lambda_2 \geq (2\nu_1)^2 = (2\lambda_1 + 2)^2.$$

According to Axiom 6.3(a) there is a non-negative increasing function $\gamma(t)$ such as-

$$\nu_n = \int_0^\infty t^n d\gamma(t) \qquad (n = 0, 1, 2, ...)$$

$$\mu_n = \int_0^\infty t^n d\gamma(t^{1/2}) \qquad (n = 0, 1, 2, ...)$$

Clearly $\beta(t^{1/2})$ is significantly different from $\gamma(t^{1/2})$, for otherwise we should have

$$\int_0^\infty t d\beta(t) = \int_0^\infty t d\gamma(t),$$

which is not possible since, ν_1 and λ_1 are unequal.

V. CONCLUSION

Any sequence which increases sufficiently rapidly leads to a soluble Stieltjes problem [with increasing $\alpha(t)$] by a slight modification of the method.The Hamburger and Stieltjes problems for the case in which $\alpha(t)$ is of closely related variation on the appropriate infinite interval. As every sequence leads to a soluble Stieltjes or Hamburger problem if we consider any function of closely related variation as a solution.condition for solving moment,determining and non-determining solution.

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Thus,

$$\mu_n = \int_0^\infty t^n d\beta(t^{1/2}) \qquad (n = 0, 1, 2, ...).$$

Next, we suppose $\{\nu_n\}_0^\infty$ be a sequence which is corresponding to the sequence $\{\lambda_n\}_0^\infty$ with an exception $\nu_1 = \lambda_1 + 1$.

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Obviously, if n is neither 1 nor 2. Then;

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According to Axiom 6.3(a) there is a non-negative increasing function $\gamma(t)$ such as-

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Study of Degree Based Topological Indices of Carbon Nanocones

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ABSTRACT: Let G be a simple and connected graph with n vertices and m edges. The degree d_u of a vertex u ∈V (G) is the number of vertices of G adjacent to u. The nanocones (CNCs) can be considered as the nanoscale conical carbon-based material. In carbon nanocones $CNC_k[n]$, the parameter k defines the length of inner cycle and n defines the number of layers of the graph. In this paper degree based topological indices for carbon nanocones $CNC_3[2]$, $CNC_4[2]$,$CNC_5[2]$,$CNC_6[2]$ and $CNC_7[2]$ and some inequalities are investigated.

KEYWORDS: Topological indices, carbon nanocones, inequality, one pentagonal nanocone, one heptagonal nanocone.

I. INTRODUCTION

A molecular graph is a simple graph such that its vertices correspond to the atoms and the edges to the bonds. A topological index is a number related to a molecular graph invariant automorphism of Graph. The ability of elemental carbon to form extended two dimensional sheet structures with extremely strong bond makes it a stable material to produce isolated objects .The sheets can be resealed notionally, to form a cone or horn [1].

Nanocones are discovered in 1994 [2].Nanocones are carbon based structures formed by introducing 60^0 disinclination defects in two-dimensional graphenes sheets [3] (fig.1).

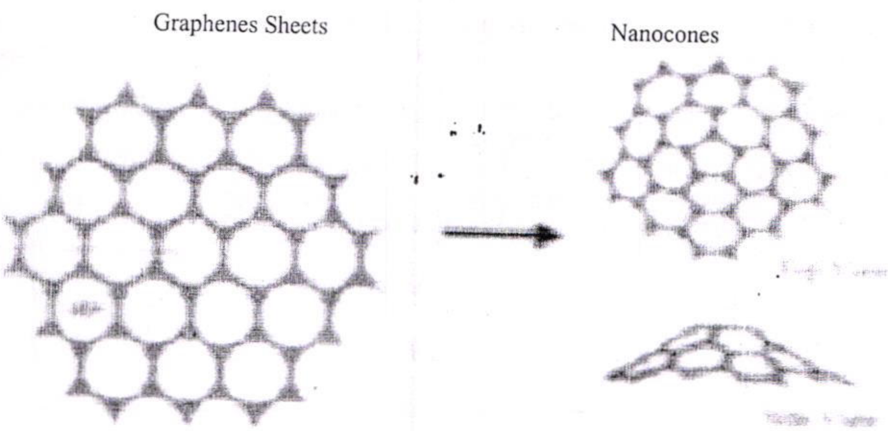

Figure (1): 2-Dimensional sheet of graphene.

Folding graphene sheets have five possible closed distinct CNC structures and the apex angles of a cone can be calculated as

Where θ is the disinclination angle in degrees can be taken as 60^0,120^0,180^0,240^0 and 360^0 [4].In carbon nanocones $CNC_k[n]$, the parameter k defines the length of the inner cycle and n defines the number of layers of the graph. The nanocones $CNC_3[2]$, $CNC_4[2]$,$CNC_5[2]$, consists of triangle ,square and pentagon as its core surrounded by two layers of hexagons respectively fig.(2). The 2-dimensional of graphof $CNC_7[1]$ withheptagon as core is also shown in fig.2. The definitions for degree based topological indices are taken from [5-10].In this paper our notations is standard and mainly taken from standard books of graph theory [11-16]. In this paper vertex degree-based topological indices:Randic index R(G), Reciprocal Randics index RR(G), general Randic indices (with = -3,-2,2,3) where is adjustable parameter, Zagreb indices(first $M_1(G)$, second $M_2(G)$ and third $M_3(G)$),modified second Zagreb index$M_2^*(G)$,secondmultiplicative Zagreb index,,modified first Zagreb indexΠ_1^*, reduced second Zagreb index$R(M_2)$, Augmented Zagreb index ABC(G),hyper Zagreb index HM(G),Atom-bond connectivity index ABC(G),Harmonic index H(G),Sum-connectivity index SCI(G),Sum-connectivity indices(with = -3,-2,2,3 where is adjustable parameter),Inverse indeg index IN(G), Geometric-Arithmetic index GA(G) and some inequalities are studied in $CNC_3[2]$, $CNC_4[2]$,$CNC_5[2]$,$CNC_6[2]$ and $CNC_7[2]$ nanocones.

II. RESULTS AND DISCUSSION

Degree-based Topological indices

In carbon nanocones $CNC_k[n]$, the parameter k defines the length of inner cycle and n defines the number of layers of the graph. The 2-dimensional graphs for $CNC_3[2]$,$CNC_4[2]$ and $CNC_5[2]$ and $CNC_7[1]$ are shown in figure (2).

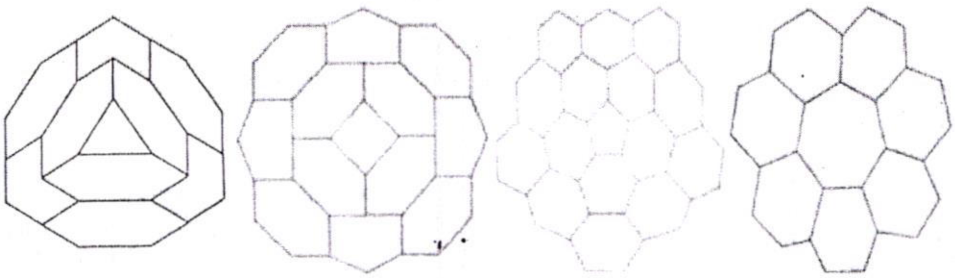

Figure (2): 2-Dimensional graphs of $CNC_3[2]$,$CNC_4[2]$,$CNC_5[2]$ and $CNC_7[1]$ nanocones

From 2-dimensional graphs of These nanocones the edges of degree (3, 3), (2, 2) and (3, 2) are counted and given in table (1).

Table no (1): The number of edges (d_u =3, d_v =3), (d_u =2, d_v = 2) and (d_u =3, d_v =2) for nanocones $CNC_k[2]$.

Carbon Nanocones	The number of edges of type d_u=3,d_v=3,(E_1)	The number of edges of type d_u=2,d_v=2 (E_2)	The number of edges of type d_u=3,d_v=2 (E_3)
$CNC_3[2]$	21	3	12
$CNC_4[2]$	29	4	16
$CNC_5[2]$	35	5	20
$CNC_6[2]$	42	6	24
$CNC_7[2]$	49	7	28

The definitions of degree-based topological indices [5 – 10] :Randic index R(G) ,Reciprocal Randic index RR(G),Randic index (with h = -3,-2,2,3) where h is adjustable parameter, Zagreb indexfirst $M_1(G)$, second $M_2(G)$, third $M_3(G)$,modified Zagreb index $M_2^*(G)$,second multiplicative Zagreb index $\prod 2$,modified first Zagreb indexΠ_1^*, reduced second Zagreb index $R(M_2)$, Augmented Zagreb index ABC(G),hyper Zagreb index HM(G),Atom-bond connectivity index ABC(G),Harmonic index H(G),Sum connectivity index SCI(G),Sum-connectivity indices with h = -3,-2,2,3 where h is adjustable parameter, Inverse indeg index IN(G), Geometric-Arithmetic index GA(G) are used for computing TIsof $CNC_3[2]$, $CNC_4[2]$,$CNC_5[2]$, $CNC_6[2]$ and $CNC_7[2]$ nanocones.

The Randic index

$R(G) = \sum_{uv \in E(G)} 1/\sqrt{d_u d_v}$

$= |E_1|/\sqrt{9} + |E_2|/\sqrt{4} + |E_3|/\sqrt{6}$

Nanocone $CNC_3[2]$

$R(G) = \sum_{uv \in E(G)} 1/\sqrt{d_u d_v} = |E_1|/\sqrt{9} + \overline{|E_2|}/\sqrt{4} + |E_3|/\sqrt{6} = 21/\sqrt{9} + 3/\sqrt{4} + 12/\sqrt{6}$

$= 13.399$

$CNC_4[2]$

$R(G) = \sum_{uv \in E(G)} 1/\sqrt{d_u d_v} = 29/\sqrt{9} + \overline{4/\sqrt{4}} + 16/\sqrt{6} = 18.2$ $CNC_5[2]$

$R(G) = \sum_{uv \in E(G)} 1/\sqrt{d_u d_v} = 35/\sqrt{9} + \overline{5/\sqrt{4}} + 20/\sqrt{6} = 22.34$ $CNC_6[2]$

$R(G) = \sum_{uv \in E(G)} 1/\sqrt{d_u d_v} = 42/\sqrt{9} + \overline{6/\sqrt{4}} + 24/\sqrt{6} = 26.79$ $CNC_7[2]$

$R(G) = \sum_{uv \in E(G)} 1/\sqrt{d_u d_v} = 49/\sqrt{9} + \overline{7/\sqrt{4}} + 28/\sqrt{6} = 31.27$

Using the number of edges of type (d_u=3,d_v=3),(d_u=2,d_v=2),(d_u=3,d_v=2) and the definitions of degree-based indices [5-10],the TIs are computed and tabled in table (2) for $CNC_3[2]$, $CNC_4[2]$,$CNC_5[2]$, $CNC_6[2]$ and $CNC_7[2]$ nanocones.

The general Randic index with h= -3 has least values due the factor × = -3 and second multiplicative Zagreb index has highest values as it is defined in multiplicationform, among the topological indices studied. By knowing number of degrees (d_u,d_v):(3,3),(2,2),(3,2) the first ,second Zagreb and Harmonic polynomials can be computed for $CNC_3[2]$, $CNC_4[2]$,$CNC_5[2]$, $CNC_6[2]$ and $CNC_7[2]$ nanocones[9,19]. The values of TIs increase from $CNC_3[2]$ to $CNC_7[2]$ nanocones due to increase in number ofedges E_1,E_2 and E_3 in $CNC_3[2]$-$CNC_7[2]$.

Inequalities for topological indices

1) H(G)≤GA(G), is obeyed for nanocones with k=3,4,5,6,7 and n=2.
2) H(G)≤X(G), is obeyed (where X(G)-Sum-connectivity index) in $CNC_3[2]$-$CNC_7[2]$.
3) The general inequality for Zagreb first and second indiceswith n vertices and m edges , $M_1(G)/n \le M_2(G)/m$ is satisfied in $CNC_4[2]$, as is the case for all graphs.

Table number (2): Topological indices of $CNC_3[2]$-$CNC_7[2]$ carbon nanocones

Topological indices	$CNC_3[2]$	$CNC_4[2]$	$CNC_5[2]$	$CNC_6[2]$	$CNC_7[2]$
R(G)	13.399	18.2	22.34	26.79	31.27
RR(G)	98.39	134.18	163.98	196.77	229.57
$R(G)\lambda = -3$	0.13123	0.176349	0.21873	0.2625	0.3063
$R(G)\lambda = -2$	0.78008	1.052532	1.3005	1.5602	1.8202
$R(G)\lambda = 2$	2181	2989	3635	4362	5089
$R(G)\lambda = 3$	18093	24853	30155	36186	42217
M_1	198	250	330	396	462
M_2	273	373	455	546	637
M_3	12	16	20	24	28
$M2^*$	5.084	6.889	8.473	10.167	11.862
Π_2	163296	400896	756000	1306368	2074464

Π_1^*	1512	2784	4200	6048	8232
$R(M_2)$	111	152	185	222	259
AZI(G)	359.21	490.33	598.69	718.4	834.15
HM(G)	1104	1508	1840	2208	2576
ABC(G)	24.61	34.18626	41.011	49.2132	57.4154
H(G)	13.3	18.07	22.17	26.6	31.34
SCI(G)	15.441	24.9975	30.7365	36.8338	43.0311
$SCI(\lambda = -3)$	0.2401	0.324759	0.4002	0.48019	0.5603
$SCI(\lambda = -2)$	1.25067	1.69562	2.0848	2.50176	2.91872
$SCI(\lambda = 2)$	1104	1508	1840	2208	2576
$SCI(\lambda = 3)$	6228	8520	10380	12456	14532
IN(G)	48.9	66.7	81.5	85.2	114.1
GA(G)	35.7578	48.6769	59.5962	71.5155	76.4347

III. CONCLUSION

The vertex degree-based topological indices are computed on 2-dimensional graphs of $CNC_3[2]$, $CNC_4[2]$,$CNC_5[2]$, $CNC_6[2]$ and $CNC_7[2]$ nanocones . By knowing number of degrees as (3,3),(2,2) and (3,2) the first ,second Zagreb polynomial and Harmonic polynomial can be computed for $CNC_3[2]$- $CNC_7[2]$ carbon nanocones.The values of vertex degree-based topological indices increase with $CNC_3[2]$-$CNC_7[2]$ carbon nanocones.

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